Ex 6A

Question 1

a) $P(X=x) = \begin{cases} 0.6, & \text{if } x=1 \\ 0.4, & \text{if } x=0 \end{cases}$

b) $\mu = 0.6$

c) $200 \times 0.6 = 120$

Question 2

a) $P(X=x) = \begin{cases} \frac{1}{2}, & \text{if } x=10 \\ \frac{1}{2}, & \text{if } x=-6 \end{cases}$

b) $\mu = \frac{1}{2}(10) + \frac{1}{2}(-6)$
$= \$2$

c) $Var(X) = \frac{1}{2}(10)^2 + \frac{1}{2}(-6)^2 - \mu^2$
$= \$64$

$\therefore \sigma = \sqrt{64}$
$= \$8$

Question 3

a) $P(X=x) = \begin{cases} \frac{1}{6}, & \text{for } x=15 \\ \frac{5}{6}, & \text{for } x=-2 \end{cases}$

b) $\mu = \frac{1}{6}(15) + \frac{5}{6}(-2) = \$\frac{5}{6}$

c) $Var(X) = \frac{1}{6}(15^2) + \frac{5}{6}(-2)^2 - \mu^2$
$= \frac{1445}{36}$

$\therefore \sigma = \frac{\sqrt{1445}}{6}$

Question 4

Since we win \$50 but it cost \$5 to enter.

a) $\mu = 0.05(45) + 0.95(-5)$
$= -\$2.50$

b) $\mu = 0.05(50-x) + 0.95(-x)$
Let $\mu = 0$
$2.5 - 0.05x - 0.95x = 0$
$x = \$2.50$

c) $\mu = p(45) + (1-p)(-5)$
Let $\mu = 0$
$45p - 5 + 5p = 0$
$\therefore p = \frac{1}{10}$

Question 5

Both correct since overall they represent the same scenario.
At a deeper level, we are using the linearity of expectation
since $E(aX+b) = aE(X) + b$

Bob's way Mary's way

Question 6

a) $\mu = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$

b) $E(x) = 1(p) + 0(1-p)$

$= p$

Question 7

a) $Var(x) = x_1^2 p_1 + x_2^2 p_2 + \cdots + x_n^2 p_n - \mu^2$

b) $Var(x) = 1^2(p) + 0^2(1-p) - \mu^2$

$= p - p^2$

Problem 1

$f(1) = p^1(1-p)^0 = p$

$f(0) = p^0(1-p)^1 = 1-p$

$$\therefore f(x) = \begin{cases} p & , \text{for } x=1 \\ 1-p & , \text{for } x=0 \end{cases}$$

This is precisely the formula for the Bernoulli distribution.

Ex 6B

Question 1

a) $\binom{3}{0}(0.2)^0(0.8)^3$

b) $\binom{3}{1}(0.2)^2(0.8)^1$

c) $\binom{3}{2}(0.2)^2(0.8)^1$

d) $\binom{3}{3}(0.2)^3(0.8)^0$

Question 2

a) $P(X \geq 7) = P(X=7) + P(X=8) = \cdots$

b) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = \cdots$

c) $P(X \geq 2) = P(X=2) + P(X=3) + \cdots + P(X=8)$

$= 1 - P(X=0) - P(X=1)$

$= \cdots$

d) $P(X \leq 7) = P(X=0) + P(X=1) + \cdots + P(X=7)$

$= 1 - P(X=8)$

$= \cdots$

Question 3 (see book)

Question 4 (see book)

Question 5 (see book)

Question 6

a) Let X = # heads flipped

$P(X=3) = \binom{5}{3}\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2$

b) $P(X \geq 1) = P(X=1) + \cdots + P(X=5)$

$= 1 - P(X=0)$

$= 1 - \binom{5}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^5$

$= 1 - \frac{1}{2^5}$

c) $P(X \geq 4) = P(X=4) + P(X=5)$

$= \binom{5}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^1 + \binom{5}{5}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^0$

Question 7

a) Let X = # of faulty toys

$n = 100$, $p = 0.02$

$P(X=0) = \binom{100}{0}(0.02)^0(0.98)^{100}$

b) $P(X \geq 2) = P(X=2) + \cdots + P(X=100)$

$= 1 - P(X=0) - P(X=1)$

$= 1 - \binom{100}{0}(0.02)^0(0.98)^{100} - \binom{100}{1}(0.02)^1(0.98)^{99}$

Question 8

a) Let $X = \#$ black balls drawn

$n = 5$, $p = \frac{6}{10} = \frac{3}{5}$

$P(X=3) = \binom{5}{3}\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^2$

b) $\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^2$

We don't include the $\binom{n}{k}$ term because the order is already given

Question 9

a) let $X = \#$ correct answers

$n = 10$, $p = \frac{1}{4}$

$P(X=0) = \binom{10}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{10}$

b) $P(X=10) = \binom{10}{10}\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^0$

c) $P(X=10) = \binom{10}{4}\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^6$

d) $P(X \geq 8) = P(X=8) + P(X=9) + P(X=10)$

$= \dots$

e) $P(X \geq 2) = P(X=2) + \dots + P(X=10)$

$= 1 - P(X=0) - P(X=1)$

$= \dots$

f) $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$= \dots$

Question 10

a) let $X = \#$ of winning tickets

$n = 4$, $p = 0.1$

$P(X=2) = \binom{4}{2}(0.1)^2(0.9)^2$

b) $P(X=4) = \binom{4}{4}(0.1)^4(0.9)^0$

c) $P(X=0) = \binom{4}{0}(0.1)^0(0.9)^4$

d) $P(X \geq 1) = P(X=1) + \dots + P(X=4)$

$= 1 - P(X=0)$

$= 1 - \binom{4}{0}(0.1)^0(0.9)^4$

Question 11

let $X = \#$ of people who drink coffee.

$n = 6$, $p = 0.4$

$P(X > 3) = P(X=4) + P(X=5) + P(X=6)$

$= \dots$

Question 12 (see book)

Question 13 (see book)

Question 14

a) let $X = \#$ of 'sixes' rolled

$n = 10$, $p = \frac{1}{6}$

$P(X=4) = \binom{10}{4}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^6$

b) $\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^6$

the order is already fully selected, so no need for the $\binom{n}{k}$ term.

c) $6\ \underbrace{x\ x\ x\ x\ x\ x\ x\ x\ x}_{\text{Still need to choose 3 more out of 9}}$ $\quad \binom{9}{3}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^6$

d) $6\ \underbrace{x\ x\ x\ x\ x\ x\ x\ x}_{\text{Still need to choose 2 more out of 8}}\ 6$ $\quad \binom{8}{2}\left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)^6$

Question 15

a) let $X = \#$ cloudy days

$n = 7,\ p = \frac{1}{5}$

$P(X=3) = \binom{7}{3}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^4$

b) $P(X=2) = \binom{7}{2}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^5$

c) X C X X X X X

cloudy Monday

$\binom{6}{4}\left(\frac{1}{5}\right)^5\left(\frac{4}{5}\right)^2$

d) X C C X X X X

Cloudy days

$\binom{5}{2}\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^3$

Question 16

a) let $X = \#$ of 'sixes' rolled

$P(X \geq 1) = P(X=1) + \dots + P(X=n)$

$= 1 - P(X=0)$

$= 1 - \binom{n}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^n$

$= 1 - \left(\frac{5}{6}\right)^n$

b) $P(X \geq 1) > 0.9$

$1 - \left(\frac{5}{6}\right)^n > 0.9$

$\left(\frac{5}{6}\right)^n < 0.1$

$n \ln\left(\frac{5}{6}\right) < \ln(0.1)$

$n > \frac{\ln(0.1)}{\ln\left(\frac{5}{6}\right)}$

the inequality flips here because $\ln\left(\frac{5}{6}\right) < 0$.

≈ 12.6

$\therefore$ 13 rolls needed.

Question 17

a) let $X = \#$ of times Bob wins

$P(X \leq 1) = P(X=0) + P(X=1)$

$= \binom{n}{0}(0.2)^0(0.8)^n + \binom{n}{1}(0.2)^1(0.8)^{n-1}$

$= 0.8^n + 0.2n(0.8)^{n-1}$

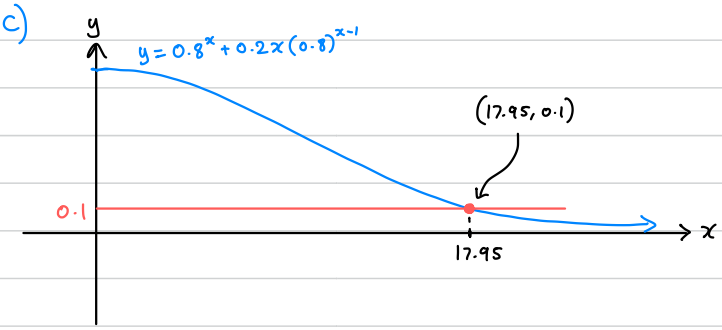
b) $P(X \geq 2) > 0.9$

$1 - P(X \leq 1) > 0.9$

$P(X \leq 1) < 0.1$

$0.8^n + 0.2n(0.8)^{n-1} < 0.1$

c)



$\therefore x > 17.95$

Least number of spins is 18.

Problem 1

Let X = # of prizes won.

$P(X \geq 1) > 0.95$

$1 - P(X=0) > 0.95$

$P(X=0) < 0.05$

$\binom{n}{0}(0.1)^0(0.9)^n < 0.05$

$n \ln(0.9) < \ln(0.05)$

$n > 28.4$

$\therefore$ Bill should spin at least 29 times.

Problem 2

Let $P(H) = p$

Let X = # of heads flipped

$P(X=4) = 9 \times P(X=2)$

$\cancel{\binom{6}{4}} p^4(1-p)^2 = 9 \times \cancel{\binom{6}{2}} p^2(1-p)^4$

$p^2 = 9(1-p)^2 \quad (p \neq 0)$

$= 9 - 18p + 9p^2$

$8p^2 - 18p + 9 = 0$

$(4p-3)(2p-3) = 0$

$p = \frac{3}{2}, \frac{3}{4}$

But $0 < p < 1$, so $p = \frac{3}{4}$.

$\therefore P(T) = \frac{1}{4}$.

Problem 3

a) Let X = # rainy days

$n = 7,\ p = \frac{1}{3}$

$P(X \geq 4) = P(X=4) + P(X=5) + P(X=6) + P(X=7)$

$= \binom{7}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^3 + \cdots + \binom{7}{7}\left(\frac{1}{3}\right)^7\left(\frac{2}{3}\right)^0$

$= \frac{379}{2187}$

b) Let X = # of mostly wet weeks

$n = 10,\ p =$ (a)

$P(X=3) = \binom{10}{3} p^3 (1-p)^7$

≈ 0.165

Problem 4

Let X = # of times Leon hits target

$P(\text{Leon passes a day}) = P(X \geq 9)$

$= P(X=9) + P(X=10)$

$\therefore p = \binom{10}{9}(0.8)^9(0.2)^1 + \binom{10}{10}(0.8)^{10}(0.2)^0$

$\underbrace{\checkmark\ \checkmark\ \checkmark\ \checkmark\ \checkmark\ \checkmark}_{\text{passes}}\ \underset{\text{eliminated here}}{\times}$

$P(\text{eliminated last day}) = p^6 \times (1-p)$, where p is defined as above.

Problem 5

Let $P(H) = p$

$\underset{\text{for 2 flips}}{P(X>1)} = \underset{\text{for 4 flips}}{P(X>2)}$

$P(X=2) = P(X=3) + P(X=4)$

$\binom{2}{2}p^2(1-p)^0 = \binom{4}{3}p^3(1-p) + \binom{4}{4}p^4(1-p)^0$

$p^2 = 4p^3(1-p) + p^4$

$= 4p^3 - 4p^4 + p^4$

$p^2 = 4p^3 - 3p^4$

$1 = 4p - 3p^2$

$3p^2 - 4p + 1 = 0$

$(3p-1)(p-1) = 0$

$p = 1, \frac{1}{3}$

But $p \neq 1$, so $p = \frac{1}{3}$.

Ex 6C

Question 1

a) $\mu = 10(0.2) = 2$

$\sigma^2 = 10(0.2)(0.8) = 1.6$

b) $\mu = 20(0.6)$

$\sigma^2 = 20(0.6)(0.4) = 4.8$

c) $\mu = 120\left(\frac{1}{3}\right) = 40$

$\sigma^2 = 120\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{80}{3}$

d) $\mu = 50\left(\frac{1}{4}\right) = 12.5$

$\sigma^2 = 50\left(\frac{1}{4}\right)\left(\frac{3}{4}\right) = 9.375$

Question 2

a) $np = 6$

$npq = 4$

$\therefore q = \frac{2}{3}$

$p = \frac{1}{3}$

$n = 18$

b) $P(X=3) = \binom{18}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^{15}$

Question 3

a) $n = 12, \ p = \frac{1}{6}$

$\mu = 12\left(\frac{1}{6}\right) = 2$

$\sigma^2 = 12\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = \frac{5}{3}$

$\sigma \approx 1.29$

b) $2 - 1.29 \leq X \leq 2 + 1.29$

$0.71 \leq X \leq 3.29$

$1 \leq X \leq 3$

$\therefore X = 1, 2, 3$

c) $P(\mu - \sigma \leq X \leq \mu + \sigma) = P(1 \leq X \leq 3)$

$= P(X=1) + P(X=2) + P(X=3)$

$= \binom{12}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{11} + \binom{12}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{10} + \binom{12}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^9$

≈ 0.763

d) See book

e) See book

Question 4

a) $\mu = 30\left(\frac{1}{2}\right) = 15$

$\sigma^2 = 30\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 7.5$

$\sigma \approx 2.74$

b) $\mu - 2\sigma \leq X \leq \mu + 2\sigma$

$9.52 \leq X \leq 20.48$

$10 \leq X \leq 20$

c) Technology

Question 5

a) $\mu = 10(\frac{1}{2}) = 5$

b) $Var(x) = 10(\frac{1}{2})(\frac{1}{2}) = 2.5$

$\sigma = \sqrt{2.5} \approx 1.58$

Question 6

a) $P(x=0) \times 2^0 + P(x=1) \times 2^1 + P(x=2) \times 2^2 + P(x=3) \times 2^3$

$= \binom{3}{0}(\frac{1}{2})^0(\frac{1}{2})^3 \times 2^0 + \cdots + \binom{3}{3}(\frac{1}{2})^3(\frac{1}{2})^0 \times 2^3$

$= \$3.375$

b) Expectation is $-\$4 + \$3.375 = -\$0.625$

$\therefore$ On average Bob will lose \$0.625 per game

$\therefore$ After 200 games, Bob will lose on average \$125

Question 7

a) let X = # of times Mary hits the target

$\mu = 5 \times 0.6 \times \$5 - \$20$ since entry fee is \$20

$= -\$5$

< 0

$\therefore$ Not in Mary's favour.

b) $\mu = 5 \times p \times \$5 - \$20$

$= 0$

$25p = 20$

$\therefore p = \frac{4}{5}$

Problem 1

a) $\cancel{\binom{5}{1}} p(1-p)^4 = \cancel{\binom{5}{4}} p^4(1-p)$

$(1-p)^3 = p^3$

$\left(\frac{1-p}{p}\right)^3 = 1$

$\frac{1-p}{p} = 1$

$\therefore p = \frac{1}{2}$

b) $\cancel{\binom{5}{2}} p^2(1-p)^3 = \cancel{\binom{5}{3}} p^3(1-p)^2$

$1-p = p$

$\therefore p = \frac{1}{2}$

c) The above imply a symmetric distribution i.e. $p = q = \frac{1}{2}$

Problem 2

Let X = # of odd digits

$n = 5$, $p = \frac{5}{9}$ 5 odd digits from 1 to 9

$P(\text{more odd than even}) = P(X \geq 3)$

$= \binom{5}{3}(\frac{5}{9})^3(\frac{4}{9})^2 + \binom{5}{4}(\frac{5}{9})^4(\frac{4}{9})^1 + \binom{5}{5}(\frac{5}{9})^5(\frac{4}{9})^0$

≈ 0.6033

$\therefore$ Expected winnings $= 0.6033 \times \$5 - \$1$ costs \$1 to play

$\approx \$2.02$

> 0

$\therefore$ Overall Mary should walk away with a profit.

Problem 3

a) $RHS = 1 - \frac{npq}{np}$

$= 1 - q$

$= p$

$= LHS$

b) $p > 0$

$1 - \frac{Var(x)}{\mu} > 0$

$\frac{Var(x)}{\mu} < 1$

$\therefore \mu > Var(x)$

c) Further, since $\frac{Var(x)}{\mu} \to 0$

d) Closer, since $\frac{Var(x)}{\mu} \to 1$

so $Var(x) \to \mu$

Ex 6D

Question 1 (see book)

Question 2

(a) - (d) (see book)

(e) $P(\hat{p}=0) = \binom{4}{0}\left(\frac{3}{5}\right)^0\left(\frac{2}{5}\right)^4$

$P(\hat{p}=0.25) = \binom{4}{1}\left(\frac{3}{5}\right)^1\left(\frac{2}{5}\right)^3$

$P(\hat{p}=0.5) = \binom{4}{2}\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^2$

$P(\hat{p}=0.75) = \binom{4}{3}\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^1$

$P(\hat{p}=1) = \binom{4}{4}\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^0$

(f) (i) $P(\hat{p}=0.5) = \binom{4}{2}\left(\frac{3}{5}\right)^2\left(\frac{2}{5}\right)^2$

(ii) $P(\hat{p}\geq 0.5) = P(\hat{p}=0.5) + P(\hat{p}=0.75) + P(\hat{p}=1)$

(iii) $P(\hat{p}\leq 0.5) = P(\hat{p}=0) + P(\hat{p}=0.25)$

Question 3

(a) - (b) (see book)

(c) $P(\hat{p}=0) = \binom{5}{0}(0.4)^0(0.6)^5$

$P(\hat{p}=0.2) = \binom{5}{1}(0.4)^1(0.6)^4$

$P(\hat{p}=0.4) = \binom{5}{2}(0.4)^2(0.6)^3$

$P(\hat{p}=0.6) = \binom{5}{3}(0.4)^3(0.6)^2$

$P(\hat{p}=0.8) = \binom{5}{4}(0.4)^4(0.6)^1$

$P(\hat{p}=1) = \binom{5}{5}(0.4)^5(0.6)^0$

(d) 0.4

(e) (i) $P(\hat{p}=0.4) = \binom{5}{2}(0.4)^2(0.6)^3$

(ii) $P(\hat{p}<\mu) = P(\hat{p}=0) + P(\hat{p}=0.2)$

(iii) $P(\hat{p}>\mu) = P(\hat{p}=0.6) + P(\hat{p}=0.8) + P(\hat{p}=1)$

Question 4 see book.

Question 5

a) $\mu = 0.4$ $\quad \sigma = \sqrt{\frac{pq}{n}}$

$= \sqrt{\frac{.4 \times .6}{20}}$

$= \sqrt{\frac{3}{250}}$

b) $0.2 = \sqrt{\frac{.2 \times .8}{n}}$

$0.04n = 0.16$

$n = 4$

c) $0.04 = \frac{\sqrt{pq}}{10}$

$0.16 = pq \Rightarrow p = 0.2, \; q = 0.8$

or $p = 0.8, \; q = 0.2$

Question 6

a) $\mu = 0.3$

b) $\sigma^2 = \frac{0.3 \times 0.7}{60}$

$= \frac{7}{2000}$

c) $\sigma = \sqrt{\frac{7}{2000}}$

Question 7

$0.1 = \sqrt{\frac{.6 \times .4}{n}}$

$0.01n = 0.24$

$n = 24$

Problem 1

(a) $P(X=k-1) = \binom{n}{k-1} p^{k-1} (1-p)^{n-k+1}$

(b) $$\frac{P(X=k)}{P(X=k-1)} = \frac{\binom{n}{k} p^{\cancel{k}} \cancel{(1-p)^{n-k}}}{\binom{n}{k-1} \cancel{p^{k-1}} (1-p)^{\cancel{n-k+1}}}$$

$$= \frac{\binom{n}{k}}{\binom{n}{k-1}} \times \frac{p}{1-p}$$

$$= \frac{\cancel{n!}}{\cancel{k!}\,\cancel{(n-k)!}} \times \frac{\overset{(n-k+1)}{\cancel{(n-k+1)!}}\,\cancel{(k-1)!}}{\cancel{n!}} \times \frac{p}{1-p}$$

$$= \frac{n-k+1}{k} \times \frac{p}{1-p}$$

(c) If $P(X=k) \geq P(X=k-1)$, then $\frac{P(X=k)}{P(X=k-1)} \geq 1$

$$\frac{n-k+1}{k} \times \frac{p}{1-p} \geq 1$$

$$p(n-k+1) \geq k(1-p)$$

$$np - \cancel{kp} + p \geq k - \cancel{kp}$$

$$\therefore k \leq np + p$$

(d) $P(X=x)$ increases in value as x increases.
The largest valid value of x is $x = np$ since
$x \leq np + p$ and $0 < p < 1$.

(e) $\hat{P}(X=x)$ is maximised when $x = np$

$\therefore \frac{x}{n} = p$

$\therefore \hat{p} = p$

Problem 2

(a) See book

(b) $$E(\hat{p}) = E\left(\frac{X}{n}\right)$$
$$= \frac{1}{n} \times E(X)$$
$$= \frac{1}{n} \times np$$
$$= p$$

(c) $$Var(\hat{p}) = Var\left(\frac{X}{n}\right)$$
$$= \frac{1}{n^2} Var(X)$$
$$= \frac{1}{n^2} \times np(1-p)$$
$$= \frac{p(1-p)}{n}$$

6E

Q1. a) $z = \dfrac{\frac{34}{40} - 0.8}{0.0632}$ $\qquad \sigma = \sqrt{\dfrac{.8 \times .2}{40}} = 0.0632$

$= 0.79$

b) $z = \dfrac{\frac{29}{40} - 0.8}{0.0632}$

$= -1.19$

Q2 a) $E(x) = 0.4 \times 160$

$= 64$

$Var(x = .4 \times .6 \times 160$

$= 38.4$

b) Large tosses

c) $z = \dfrac{\frac{70}{160} - 0.4}{0.0387}$ $\qquad \sigma = \sqrt{\dfrac{.4 \times .6}{160}}$

$= 0.97$

d) $P(x \leq 70) = P(z \leq 0.97)$

$= 0.8340$

e) $P(x \geq 72) = P\left(z > \dfrac{\frac{72}{160} - 0.4}{0.0387}\right)$

$= P(z > 1.29)$

$= 1 - 0.9015$

$= .0985$

f) $P(x \leq 62) = P\left(z \leq \dfrac{\frac{62}{160} - 0.4}{0.0387}\right)$

$= P(z < -0.323)$

$= 1 - 0.6255$

$= 0.3745$

Q3 a) $\bar{x}_{\hat{p}} = 0.15$ $\qquad \sigma_{\hat{p}} = \sqrt{\dfrac{.15 \times .85}{120}} = 0.0326$

b) $P(x \leq 24) = P\left(z \leq \dfrac{\frac{24}{120} - 0.15}{0.0326}\right)$

$= P(z \leq 1.534)$

$= 0.9370$

c) $P(x \leq 15) = P\left(z \leq \dfrac{\frac{15}{120} - 0.15}{0.0326}\right)$

$= P(z \leq -0.77)$

$= 1 - 0.7794$

$= 0.22$

Q4 a) $E(X) = 0.6 \times 100 = 60$ $\qquad Var(X) = 60 \times 0.4 = 24$

$\sigma = \sqrt{\dfrac{.6 \times .4}{100}} = 0.049$

b) $P(62 < x < 66) = P\left(\dfrac{0.62 - 0.6}{0.049} < z < \dfrac{0.66 - 0.6}{0.049}\right)$

$= P(0.41 < z < 1.22)$

$= 0.8888 - 0.6591$

$= 0.2297$

c) $P(55 < x < 62) = P\left(\dfrac{0.55 - 0.6}{0.049} < z < 0.41\right)$

$= P(-1.02 < z < 0.41)$

$= 0.6591 - (1 - 0.8461)$

$= 0.5052$

d) $P(50 < x < 55) = P\left(\frac{0.5-0.6}{0.049} < z < -1.02\right)$

$= P(-2.04 < z < -1.02)$

$= P(1.02 < z < 2.04)$

$= 0.9793 - 0.8461$

$= 0.1332$

Q5 a) $\mu = 0.1 \quad \sigma = \sqrt{\frac{.1 \times .9}{90}} = 0.0316$

b) $\hat{p} = \frac{12}{90} = \frac{2}{15}$

c) $z = 1.055$

d) $P(X > 12) = 1 - 0.8531$

$= 0.1469$

Q6 a) $\mu = 0.8 \quad \sigma = \sqrt{\frac{.8 \times .2}{50}} = 0.05657$

$P(x > 85\%) = P\left(z > \frac{0.85-0.8}{0.05657}\right)$

$= P(z > .8839)$

$= 1 - 0.8106$

$= 0.1894$

b) $P(X \leq 36) = P\left(z < \frac{\frac{36}{50} - 0.8}{0.5657}\right)$

$= P(z \leq -1.414)$

$= 1 - 0.9207$

$= 0.0793$

Q7 a) $\mu = 0.56 \quad \sigma = \sqrt{\frac{0.56 \times 0.44}{160}} = 0.03924$

$P(\hat{p} > 0.62) = P\left(z > \frac{0.62-0.56}{0.03924}\right)$

$= P(z > 1.529)$

$= 1 - 0.9370$

$= 0.063$

b) $P(0.52 < \hat{p} < 0.62) = P\left(\frac{.52-.56}{0.03924} < z < 1.53\right)$

$= P(-1.02 < z < 1.53)$

$= 0.9370 - (1 - .8461)$

$= 0.7831$

Q8 $P(X \geq 225) = P\left(z \geq \frac{.75-.8}{\sigma}\right) \qquad \sigma = \sqrt{\frac{.8 \times .2}{300}}$

$= P(z \geq -2.165) \qquad = 0.0231$

$= P(z \leq 2.165) = 0.9846$

Q9 $\mu = 0.4 \quad \sigma = \sqrt{\frac{.4 \times .6}{100}} = 0.04899$

$P(35 \leq x \leq 45) = P\left(\frac{.35-.4}{\sigma} \leq z \leq \frac{.45-.4}{\sigma}\right)$

$= P(-1.02 \leq z \leq 1.02)$

$= (0.8461 - .5) \times 2$

$= 0.6922$

P1.

$$\sigma = \sqrt{\frac{.24 \times .76}{800}} = 0.0151$$

$$P(21\% < x < 23\%) = P\left(\frac{.21 - .24}{\sigma} < z < \frac{.23 - .24}{\sigma}\right)$$

$$= P(-1.99 < z < -0.66)$$

$$= P(.66 < z < 1.99)$$

$$= 0.9767 - 0.7454$$

$$= 0.2313$$

P2

$$\mu = .55 \qquad \sigma = \sqrt{\frac{.55 \times .45}{80}} = 0.05562$$

$$P(36 < x < 46) = P\left(\frac{\frac{36}{80} - .55}{\sigma} < z < \frac{\frac{46}{80} - .55}{\sigma}\right)$$

$$= P(-1.80 < z < 0.45)$$

$$= 0.6736 - (1 - .9641)$$

$$= 0.6377$$

Ex 6 Review

Question 1

(a) $P(X=x) = \begin{cases} 3, & \text{if } x = \frac{1}{6} \\ -1, & \text{if } x = \frac{5}{6} \end{cases}$

(b) $E(X) = \frac{1}{6}(3) + \frac{5}{6}(-1)$
$= -\frac{1}{3}$

(c) $30 \times \$-\frac{1}{3} = -\$10$
$\therefore$ Lose \$10 on average

Question 2

(a) $\mu = 0.1(17) + 0.9(-3)$
$= -1$
< 0
$\therefore$ Not favourable

(b) $\mu = 0.1(20-x) + 0.9(-x)$
$= 2 - x$
Let $\mu = 0$
$x = 2$
$\therefore$ \$2 to be fair

(c) $\mu = p(17) + (1-p)(-3)$
$= 17p - 3 + 3p$
$= 20p - 3$
Let $\mu = 0$
$\therefore p = \frac{3}{20}$

Question 3

(a)

(b)

(c)

(d)

Question 4

(a) $P(X \geq 5) = P(X=5) + P(X=6)$
$= \binom{6}{5}(0.2)^5(0.8)^1 + \binom{6}{6}(0.2)^6$

(b) $P(X \leq 1) = P(X=0) + P(X=1)$
$= \binom{6}{0}(0.8)^6 + \binom{6}{1}(0.2)^1(0.8)^5$

(c) $P(X \geq 1) = 1 - P(X=0)$
$= 1 - \binom{6}{0}(0.8)^6$

(d) $P(X \leq 5) = 1 - P(X=6)$
$= 1 - \binom{6}{6}(0.2)^6$

Question 5

(a) $n = 4,\ p = \frac{1}{6}$

$P(x=3) = \binom{4}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^1$

(b) $P(x \geq 1) = 1 - P(x=0)$

$= 1 - \binom{4}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^4$

(c) $P(x \geq 3) = P(x=3) + P(x=4)$

$= \binom{4}{3}\left(\frac{1}{6}\right)^3\left(\frac{5}{6}\right)^1 + \binom{4}{4}\left(\frac{1}{6}\right)^4$

Question 6

(a) $n = 5,\ p = \frac{1}{4}$

$P(x=0) = \binom{5}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^5$

(b) $P(x=5) = \binom{5}{5}\left(\frac{1}{4}\right)^5$

(c) $P(x=4) = \binom{5}{4}\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^1$

(d) $P(x \geq 4) = P(x=4) + P(x=5)$

$= \binom{5}{4}\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right) + \binom{5}{5}\left(\frac{1}{4}\right)^5$

(e) $P(x \geq 1) = 1 - P(x=0)$

$= 1 - \binom{5}{0}\left(\frac{1}{4}\right)^5$

(f) $P(x \leq 1) = P(x=0) + P(x=1)$

$= \binom{5}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^5 + \binom{5}{1}\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^4$

Question 7

(a) $n = 6,\ p = \frac{2}{3}$

$P(x=4) = \binom{6}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2$

(b) H H H H T T

$\binom{4}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2$

(c) H _ _ _ _ _

$\binom{5}{3}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2$

(d) H H H _ _ _

$\binom{3}{1}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2$

Question 8

(a) let x = # of 6's rolled

$P(x \geq 1) = 1 - P(x=0)$

$= 1 - \binom{n}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^n$

$= 1 - \left(\frac{5}{6}\right)^n$

(b) $P(x \geq 1) > 0.99$

$1 - \left(\frac{5}{6}\right)^n > 0.99$

$\left(\frac{5}{6}\right)^n < 0.01$

$n \ln\left(\frac{5}{6}\right) < \ln(0.01)$

$n > \frac{\ln(0.01)}{\ln\left(\frac{5}{6}\right)}$

Question 9

Let X = # of 'double sixes' rolled.

$P(X \geq 1) > 0.5$

$1 - \left(\frac{35}{36}\right)^n > 0.5$

$\left(\frac{35}{36}\right)^n < 0.5$

$n \ln \frac{35}{36} < \ln 0.5$

$n > \frac{\ln(0.5)}{\ln \frac{35}{36}}$

> 24.6

$n = 25$

Question 10

a) Let P = Prob of 2 boys, 2 girls

$= \binom{4}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2$

$= \frac{3}{8}$

b) Let X = # of families with 2B, 2G.

$\binom{5}{2}\left(\frac{3}{8}\right)^2\left(\frac{5}{8}\right)^3$

c) None or 1 family.

$\binom{5}{0}\left(\frac{3}{8}\right)^0\left(\frac{5}{8}\right)^5 + \binom{5}{1}\left(\frac{3}{8}\right)^1\left(\frac{5}{8}\right)^4$

Question 11

a) $\frac{4}{8} \times \frac{3}{7} = \frac{3}{14}$

b) $\binom{5}{3}\left(\frac{3}{14}\right)^3\left(\frac{11}{14}\right)^2$

Question 12

a) $E\left(\frac{1}{3}X^2 - 2\right) = \frac{1}{3}E(X^2) - 2$

$\sigma^2 = E(X^2) - \mu^2$

$npq = E(X^2) - \mu^2$

$E(X^2) = npq + \mu^2$

$= npq + (np)^2$

$= 4 \times \frac{1}{3} \times \frac{2}{3} + \left(4 \times \frac{1}{3}\right)^2$

$= \frac{8}{9} + \frac{16}{9}$

$= \frac{24}{9}$

$\therefore E\left(\frac{1}{3}X^2 - 2\right) = \frac{1}{3} \times \frac{24}{9} - 2$

$= -\frac{10}{9}$

b) $30 \times \$-\frac{10}{9} = -\frac{\$100}{3}$

Bob should expect to lose $\$\frac{100}{3}$ after 30 games.

Question 13

a) $P(\hat{p} = 0.2) = \binom{5}{1}(0.4)(0.6)^4$

b) $P(\hat{p} = 0.4) = \binom{5}{2}(0.4)^2(0.6)^3$

c) $P(\hat{p} = 0.8) = \binom{5}{4}(0.4)^4(0.6)$.

Question 14

a) $\mu = np = 8 \quad Var(X) = npq = \frac{24}{5}$

b) $\mu = np = 10 \quad Var(X) = npq = 9$

c) $\mu = np = 60 \quad Var(X) = npq = 20$

Question 15

$np = 12$
$npq = 9 \implies q = \frac{3}{4}, \; p = \frac{1}{4} \quad n = 48$

Question 16

10L 6C

$P(L) = .12 \qquad P(C) = 0.045$

$P(3L) = {}^{10}C_3 (.12)^3 (.88)^7$

$= 0.08474$

$P(2C) = {}^{6}C_2 (0.045)^2 (.955)^4$

$= 0.0253$

$P(3L) \times P(2C) = 0.00214$

Question 17

a) $E(X) = 0.8 \times 10 \times 10 - 60 = \$20$ in Elle's favour
+ve

b) $E(X) = 0.8 \times 9 \times 10 - 60 = \$12$
still favourable

Question 18

$\sigma < 0.05 \qquad \sqrt{\frac{\frac{1}{2} \times \frac{1}{2}}{n}} < 0.05$

$\frac{1}{4n} < \frac{1}{400}$

$n > 100$

Question 19

a) $n = 20 \quad \hat{p} = .5 \quad \frac{pq}{n} = \frac{.5 \times .5}{20}$

$P(X > 10) = P(Z > 0)$

$= 0.5$

b) $P(X \leq 16) = P\left(Z < \frac{\frac{16}{20} - \frac{10}{20}}{0.1118}\right) \qquad \sigma = \sqrt{\frac{.5 \times .5}{20}}$

$= 0.1118$

$= P(Z < 2.68)$

$= 0.996$

d) $P(x<6) = P\left(z < \frac{.3-.5}{0.1118}\right)$

$= P(z < -1.7889)$

$= 1 - P(z < 1.79)$

$= 1 - 0.9633$

$= 0.0367$

Question 20

a) $E(\hat{p}) = 0.16 \qquad Var(\hat{p}) = \frac{.16 \times .84}{200} = \frac{21}{31250}$

b) $P(X<28) = P\left(z < \frac{0.14-0.16}{\sigma}\right)$

$= P(z < -0.772)$

$= 1 - P(z < 0.772)$

$= 1 - 0.7794$

$= 0.2206$

c) $P(24<x<36) = P\left(\frac{.12-.16}{\sigma} < z < \frac{.18-.16}{\sigma}\right)$

$= P(-1.54 < z < 0.77)$

$= 0.7794 - (1 - .9382)$

$= 0.7176$

Question 21

$\sigma = \sqrt{\frac{.65 \times .35}{40}} = 0.0754$

$P(22 < x < 24)$

$= P\left(\frac{\frac{22}{40} - .65}{\sigma} < z < \frac{\frac{24}{40} - .65}{\sigma}\right)$

$= P(-1.33 < z < -0.66)$

$= P(0.66 < z < 1.33)$

$= .9082 - .7454$

$= 0.1628$