4A

Q5 a) $\sin \frac{\pi}{2} = 1$

b) $\cos \frac{\pi}{2} = 0$

c) Let $\cos^{-1} \frac{2\sqrt{2}}{3} = A$

$\cos A = \frac{2\sqrt{2}}{3}$

$\therefore \cos\left(\cos^{-1} \frac{2\sqrt{2}}{3}\right) = \cos A = \frac{2\sqrt{2}}{3}$

d) $\sin A = \frac{1}{3}$ so

$\sin\left(\cos^{-1} \frac{2\sqrt{2}}{3}\right) = \sin A = \frac{1}{3}$

Q6 a) $\sin \frac{5\pi}{3} = \sin\left(\pi + \frac{2\pi}{3}\right) = -\sin \frac{2\pi}{3}$
$= -\frac{\sqrt{3}}{2}$

b) $\tan\left(\pi - \frac{\pi}{3}\right) = -\sqrt{3}$

c) $\cos\left(\pi + \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

d) $\sin\left(\pi + \frac{\pi}{6}\right) = -\frac{1}{2}$

e) $\cos\left(\pi - \frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

f) $-\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

g) $-\tan \frac{5\pi}{6} = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$

h) $\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

i) $\sin \frac{5\pi}{6} = \sin \frac{\pi}{6} = \frac{1}{2}$

Q7 a) $\cos^{-1}\left(-\sin \frac{\pi}{3}\right) = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
$= \pi - \cos^{-1} \frac{\sqrt{3}}{2} = \pi - \frac{\pi}{6}$
$= \frac{5\pi}{6}$

b) $\tan^{-1}\left(-\tan \frac{\pi}{6}\right) = -\frac{\pi}{6}$

c) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

Q8

Q9 a) $\frac{\pi}{5}$

b) $\frac{3\pi}{5}$

c) $\frac{\pi}{7}$

d) $\sin^{-1}\left(-\sin \frac{\pi}{5}\right) = -\sin^{-1} \sin \frac{\pi}{5} = -\frac{\pi}{5}$

e) $\cos^{-1} \cos \frac{\pi}{7} = \frac{\pi}{7}$

f) $\tan^{-1}\left(-\tan \frac{\pi}{5}\right) = -\tan^{-1}\left(\tan \frac{\pi}{5}\right) = -\frac{\pi}{5}$

g) $\cos^{-1}\left(-\cos \frac{\pi}{5}\right) = \pi - \cos^{-1} \cos \frac{\pi}{5}$
$= \pi - \frac{\pi}{5} = \frac{4}{5}\pi$

h) $\sin^{-1} -\sin \frac{4\pi}{7} = -\sin^{-1} \sin \frac{3\pi}{7} = -\frac{3\pi}{7}$

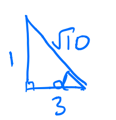
i) $\tan^{-1} \tan \frac{\pi}{5} = \frac{\pi}{5}$

Q10 a) 1

b) $\tan\left(-\tan^{-1} \frac{3}{5}\right) = -\tan \tan^{-1} \frac{3}{5} = -\frac{3}{5}$

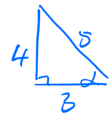
c) $\cos\left(\pi - \cos^{-1} \frac{1}{2}\right) = -\cos \cos^{-1} \frac{1}{2} = -\frac{1}{2}$.

Q11 a) $\cos \alpha = \frac{3}{\sqrt{10}}$



b) $-\tan \sin^{-1}\left(\frac{1}{5}\right) = -\tan \alpha$
$= -\frac{1}{2\sqrt{6}}$

$\tan(\pi - \cos^{-1}\frac{3}{5})$
$= -\tan\cos^{-1}\frac{3}{5}$
$= -\frac{4}{3}$



$\sin(\pi - \cos^{-1}\frac{3}{5})$
$= \sin\cos^{-1}\frac{3}{5} = \frac{4}{5}$

Q12 a) $\cos(0 - \frac{\pi}{3}) = \cos\frac{\pi}{3} = \frac{1}{2}$

b) $\tan(\frac{\pi}{2} + \frac{\pi}{6}) = \tan\frac{2\pi}{3} = -\sqrt{3}$

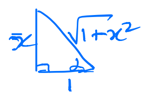
c) $\cos(\pi - \frac{\pi}{3} + \frac{\pi}{6}) = \cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$

d) $\sin(-\frac{\pi}{6} + \frac{\pi}{3}) = \frac{1}{2}$

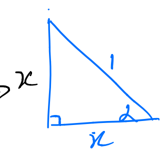
Q13 a) $x = 1 + \cos y$
$y = \cos^{-1}(x-1)$

b) $x = 1 - 2\sin\frac{y}{3}$
$\sin\frac{y}{3} = \frac{1-x}{2}$
$y = 3\sin^{-1}(\frac{1-x}{2})$

Q14 let $\cos^{-1}y = \alpha \Rightarrow \cos\alpha = y$
$\therefore x = \tan\alpha$
$\therefore y = \frac{1}{\sqrt{1+x^2}}$



Q15 let $\cos^{-1}x = \alpha$
$\sin^{-1}x = \alpha$
$\sin\alpha = x$
$\therefore \alpha = \frac{\pi}{4}$



Q16 a) $A = \sin^{-1}x$

b) $B = \cos^{-1}x$

c) $A + B = \frac{\pi}{2}$ from the diagram
$\therefore \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$.

d) if $x = \frac{1}{2}$, $A = \frac{\pi}{6}$ and $B = \frac{\pi}{3}$
and $\frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$

P1 a) $y \geq 1$

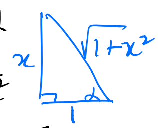
b) $x \geq 1$
$y = \operatorname{cosec} x$ inverse is $x = \operatorname{cosec} y$
$\frac{1}{x} = \sin y$
$\therefore y = \sin^{-1}\frac{1}{x}$

P2 a) $\tan^{-1}(\tan(\frac{\pi}{2} - x)) = \frac{\pi}{2} - x$

b) $\sin^{-1}(\sin(\frac{\pi}{2} - x)) = \frac{\pi}{2} - x$

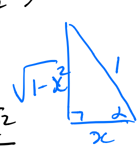
c) $\cos^{-1}(\cos(\frac{\pi}{2} - x)) = \frac{\pi}{2} - x$

P3 a) let $\tan^{-1}x = \alpha$
$\therefore \cos\alpha = \frac{1}{\sqrt{1+x^2}}$

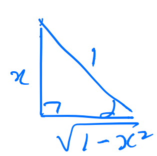


b) $\sin\alpha = \frac{x}{\sqrt{1+x^2}}$

c) let $\cos^{-1}x = \alpha$
$\tan\alpha = \frac{\sqrt{1-x^2}}{x}$



d) let $\sin^{-1}x = \alpha$
$\tan\alpha$
$= \frac{x}{\sqrt{1-x^2}}$



4B

4C

Q1 a) $\cos 2A = 1 - 2\sin^2 A$

b) $\sin A = \frac{4}{5}$

c) $\therefore 1 - 2\sin^2 A = 1 - 2\left(\frac{4}{5}\right)^2$

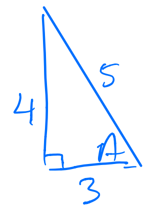
$= -\frac{7}{25}$

Q2 a) Let $\cos^{-1}\frac{3}{5} = A$

$\therefore \sin\left(2\cos^{-1}\frac{3}{5}\right) = \sin 2A$

$= 2\sin A\cos A$

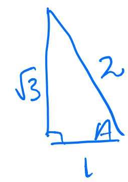
$= 2 \times \frac{4}{5} \times \frac{3}{5} = \frac{24}{25}$



b) Let $\tan^{-1}\sqrt{3} = A$

$\therefore \sin 2A = 2\sin A\cos A$

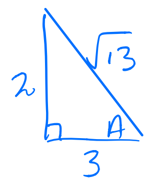
$= 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$



c) Let $\tan^{-1}\frac{2}{3} = A$

$\therefore \cos 2A = 2\cos^2 A - 1$

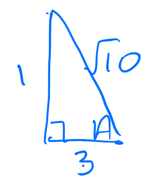
$= 2 \times \frac{9}{13} - 1 = \frac{5}{13}$



d) Let $\tan^{-1}\frac{1}{3} = A$

$\therefore \sin 2A = 2\sin A\cos A$

$= 2 \times \frac{1}{\sqrt{10}} \times \frac{3}{\sqrt{10}} = \frac{3}{5}$



e) Let $\sin^{-1}\frac{1}{3} = A$

$\therefore \tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

$= \frac{2 \times \frac{1}{2\sqrt{2}}}{1 - \frac{1}{8}} = \frac{\frac{8}{\sqrt{2}}}{7} = \frac{4\sqrt{2}}{7}$

f) Let $\cos^{-1}\frac{2}{5} = A$

$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$

$= \frac{2 \times \frac{\sqrt{21}}{2}}{1 - \frac{21}{4}} = -\frac{4\sqrt{21}}{17}$

Q3 a) $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

b) $\tan A = \frac{12}{5}$ $\tan B = \frac{3}{4}$

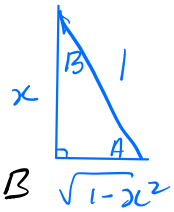
c) $\tan(A - B) = \frac{\frac{12}{5} - \frac{3}{4}}{1 + \frac{12}{5} \times \frac{3}{4}} = \frac{48 - 15}{20 + 36}$

$= \frac{33}{56}$

d) $\tan^{-1}\frac{33}{56}$

Q4 a) Let $\tan^{-1}\dfrac{x}{\sqrt{1-x^2}} = A$

and $\cos^{-1}x = B$



$\cos(A+B)$

$= \cos A\cos B - \sin A\sin B$

$= \sqrt{1-x^2}\,(x) - (x)\sqrt{1-x^2} = 0$

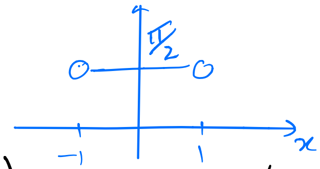
b) Since $\cos(A+B) = 0$

$A+B = \frac{\pi}{2}$

ie $\tan^{-1}\dfrac{x}{\sqrt{1-x^2}} + \cos^{-1}x = \frac{\pi}{2}$

true if $-1 < x < 1$

c)



Q5 a) Let $A = \tan^{-1}4$ $\quad B = \tan^{-1}\frac{3}{5}$

$\therefore \tan A = 4 \qquad \tan B = \frac{3}{5}$

$\tan(A-B) = \dfrac{4-\frac{3}{5}}{1+4\times\frac{3}{5}} = \dfrac{17}{17} = 1$

$\therefore A - B = \frac{\pi}{4}$

b) Let $\tan^{-1}\frac{1}{2} = A$ $\quad \tan^{-1}\frac{1}{3} = B$

$\tan(A+B) = \dfrac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}} = 1$

$\therefore A+B = \frac{\pi}{4}$

ie. $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \frac{\pi}{4}$

c) Let $\tan^{-1}2 = A$ $\quad \tan^{-1}3 = B$

$\tan(A+B) = \dfrac{2+3}{1-2\times 3} = -1$

$\therefore A+B = \frac{3\pi}{4}$

ie $\tan^{-1}2 + \tan^{-1}3 = \frac{3\pi}{4}$

Let $\sin^{-1}\frac{3}{5} = A$ $\quad \sin^{-1}\frac{12}{13} = B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$= \frac{4}{5}\times\frac{5}{13} - \frac{3}{5}\times\frac{12}{13} = \frac{-16}{65}$

$\therefore A+B = \cos^{-1}\left(\frac{-16}{65}\right)$

ie $\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{12}{13} = \cos^{-1}\left(\frac{-16}{65}\right)$

Q6 a) Let $\cos^{-1}x = A \Rightarrow \cos A = x$

$\cos 2A = 2\cos^2 A - 1$

$= 2x^2 - 1$

$\therefore 2A = \cos^{-1}(2x^2 - 1)$

ie. $2\cos^{-1}x = \cos^{-1}(2x^2-1)$

b) Let $\tan^{-1}x = A$

$\therefore \tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

$= \dfrac{2x}{1-x^2}$

$\therefore 2A = \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)$

ie $2\tan^{-1}x = \tan^{-1}\dfrac{2x}{1-x^2}$

c) Let $\sin^{-1}x = A$

$\sin 2A = 2\sin A\cos A$

$= 2x\sqrt{1-x^2}$

$\therefore 2A = \sin^{-1}(2x\sqrt{1-x^2})$

ie $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$

d) Let $\cos^{-1}x = A$

$\sin 2A = 2\sin A\cos A$

$= 2\sqrt{1-x^2} \times x$

$\therefore 2A = \sin^{-1}(2x\sqrt{1-x^2})$

ie. $2\cos^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$

Q7 Let $\angle BAO = \alpha$.

$\therefore \tan(\alpha+\theta) = \dfrac{8}{x}$ $\tan\alpha = \dfrac{2}{x}$

$\dfrac{\tan\alpha + \tan\theta}{1-\tan\alpha\tan\theta} = \dfrac{8}{x}$

$\dfrac{\frac{2}{x} + \tan\theta}{1 - \frac{2}{x}\tan\theta} = \dfrac{8}{x}$

$\dfrac{2 + x\tan\theta}{x - 2\tan\theta} = \dfrac{8}{x}$

$2x + x^2\tan\theta = 8x - 16\tan\theta$

$\tan\theta = \dfrac{6x}{x^2+16}$

$\therefore \theta = \tan^{-1}\dfrac{6x}{x^2+16}$

Q8 a) Let $\cos^{-1}x = A$ $\sin^{-1}x = B$

$\therefore \sin(A-B) = \sin\frac{\pi}{4}$

$\sin A\cos B - \cos A\sin B = \frac{1}{\sqrt{2}}$

$\sqrt{1-x^2}\sqrt{1-x^2} - x(x) = \frac{1}{\sqrt{2}}$

$1 - x^2 - x^2 = \frac{1}{\sqrt{2}}$

$x^2 = \frac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right)$

$= \frac{1}{4}(2-\sqrt{2})$

$\therefore x = \frac{1}{2}\sqrt{2-\sqrt{2}}$

b) now $\cos^{-1}x - \sin^{-1}x = \frac{\pi}{4}$

$\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$

$2\cos^{-1}x = \frac{3\pi}{4}$

$\cos^{-1}x = \frac{3\pi}{8}$

$x = \cos\frac{3\pi}{8}$

or $2\sin^{-1}x = \frac{\pi}{4}$

$x = \sin\frac{\pi}{8}$

c) true since $x = \frac{\sqrt{2-\sqrt{2}}}{2} = \cos\frac{3\pi}{8}$

P1 Let $B = \tan^{-1}2$ $\quad C = \tan^{-1}3$

$\tan(B+C) = \frac{2+3}{1-6} = -1$

$\therefore B + C = \tan^{-1}(-1) = \frac{3\pi}{4}$

$\therefore \tan^{-1}1 + \tan^{-1}2 + \tan^{-1}3$

$= \frac{\pi}{4} + \frac{3\pi}{4} = \pi$

P1 a) Let $\tan^{-1}\frac{4}{5} = A$ $\quad \tan^{-1}\frac{5}{4} = B$

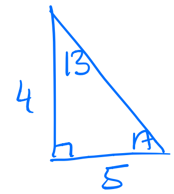
$\tan(A+B) = \dfrac{\frac{4}{5} + \frac{5}{4}}{1 - \frac{4}{5}\times\frac{5}{4}} = \frac{41}{0}$

$\therefore$ is undefined

So $A + B = \frac{\pi}{2}$

ie $\tan^{-1}\frac{4}{5} + \tan^{-1}\frac{5}{4} = \frac{\pi}{2}$

b)



c) they are reciprocals

P3. a) Let $\sin^{-1}\frac{3}{5} = A$ $\quad \sin^{-1}\frac{4}{5} = B$

$\sin(A+B) = \sin A\cos B + \cos A \sin B$

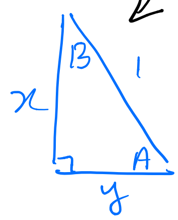
$= \frac{3}{5}\times\frac{3}{5} + \frac{4}{5}\times\frac{4}{5} = 1$

$\therefore \sin^{-1}\frac{3}{5} + \sin^{-1}\frac{4}{5} = \frac{\pi}{2}$

b) If $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$

these are complimentary angles in a $\triangle$

$\therefore x^2 + y^2 = 1$



c) same argument

REVIEW

Q1

a) $\frac{\pi}{4}$

b) $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$

c) $-\frac{\pi}{4}$

d) $\sqrt{3}$

e) $\cos(-\frac{\pi}{2}) = 0$

f) $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

g) $\cos\frac{\pi}{3} = \frac{1}{2}$

h) $\cos(\pi - \frac{\pi}{3}) = -\frac{1}{2}$

i) $\sin(-\frac{\pi}{6}) = -\frac{1}{2}$

Q2

a) $\frac{\pi}{3}$

b) $\tan^{-1}(\tan\frac{\pi}{5}) = \frac{\pi}{5}$

c) $\cos^{-1}(-\frac{1}{\sqrt{2}}) = \frac{3\pi}{4}$

d) $\sin^{-1}(-\sin\frac{\pi}{4}) = -\sin^{-1}(\sin\frac{\pi}{4})$
$= -\frac{\pi}{4}$

or $\sin^{-1}(-\frac{1}{\sqrt{2}}) = -\frac{\pi}{4}$

e) $\sin^{-1}(-\sin\frac{\pi}{8}) = -\frac{\pi}{8}$

f) $\cos^{-1}(-\cos\frac{\pi}{8}) = \pi - \cos^{-1}\cos\frac{\pi}{8}$
$= \frac{7\pi}{8}$

Q3

a) let $\cos^{-1}(\frac{3}{4}) = A$
$\sin 2A = 2\sin A\cos A$
$= 2\frac{\sqrt{7}}{4} \times \frac{3}{4} = \frac{6\sqrt{7}}{16}$

b) Let $\sin^{-1}\frac{1}{3} = A$
$\cos 2A = 1 - 2\sin^2 A$
$= 1 - \frac{2}{9} = \frac{7}{9}$

c) Let $\tan^{-1}\frac{1}{2} = A$
$\sin 2A = 2\sin A\cos A$
$= 2 \times \frac{1}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{4}{5}$

Q4

a) $\cos 2x = 1 - 2\sin^2 x$
$= 1 - 2 \times \frac{4}{25} = \frac{17}{25}$

b) $\sin 2x = 2\sin x\cos x$
$= 2 \times \frac{2}{5} \times \frac{\sqrt{21}}{5} = \frac{4\sqrt{21}}{25}$

c) $\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2 \times \frac{2}{\sqrt{21}}}{1 - \frac{4}{21}}$
$= \frac{4\sqrt{21}}{17}$

or $\tan 2x = \frac{\sin 2x}{\cos 2x}$
$= \frac{4\sqrt{21}}{25} \div \frac{17}{25} = \frac{4\sqrt{21}}{17}$

Q5 a) Let $\arctan\frac{2}{3} = A$
$\therefore \tan A = \frac{2}{3}$
Now $\tan 2A = \frac{2\tan A}{1-\tan^2 A}$
$= \frac{2 \times \frac{2}{3}}{1 - \frac{4}{9}} = \frac{12}{5}$

$\therefore\ 2A = \tan^{-1}\frac{12}{5}$

i.e. $2\arctan\frac{2}{3} = \arctan\frac{12}{5}$

b) Let $\arccos\frac{3}{5} = A$ $\quad \arctan\frac{3}{4} = B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$= \frac{4}{5}\times\frac{4}{5} - \frac{3}{5}\times\frac{3}{5} = \frac{7}{25}$

$\therefore\ A - B = \arcsin\frac{7}{25}$

i.e. $\arccos\frac{3}{5} - \arctan\frac{3}{4} = \arcsin\frac{7}{25}$

Q6. a) Let $\sin^{-1}\frac{2}{9} = A$ $\quad \cos^{-1}\frac{2}{9} = B$

Consider $\sin(A+B)$

$= \sin A\cos B - \cos A\sin B$

$= \frac{2}{9}\times\frac{2}{9} + \frac{\sqrt{77}}{9}\times\frac{\sqrt{77}}{9} = 1$

$\Rightarrow\ A + B = \frac{\pi}{2}$

b) Let $\tan^{-1}\frac{1}{4} = A$ $\quad \tan^{-1}\frac{3}{5} = B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$= \frac{\frac{1}{4} + \frac{3}{5}}{1 - \frac{1}{4}\times\frac{3}{5}} = \frac{17}{17} = 1$

$\therefore\ A + B = \frac{\pi}{4}$

We know $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$

$\therefore\ LHS = \cos^{-1}\left(\frac{2}{5}\right) + \pi - \cos^{-1}\frac{2}{5}$

$= \pi = RHS$.

Let $\tan^{-1}\frac{5}{12} = A$ $\quad \cos^{-1}\frac{5}{13} = B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$= \frac{12}{13}\times\frac{5}{13} - \frac{5}{13}\times\frac{12}{13} = 0$

$\therefore\ A + B = \frac{\pi}{2}$.

Q7 a) Let $\sin^{-1}\frac{1}{2} = D$ $\quad \sin^{-1}\frac{1}{4} = B$

$\sin(D-B) = \sin D\cos B - \cos D\sin B$

$= \frac{1}{2}\times\frac{\sqrt{15}}{4} - \frac{\sqrt{3}}{2}\times\frac{1}{4}$

$= \frac{1}{8}(\sqrt{15} - \sqrt{3})$

$\therefore\ A = \frac{1}{8}(\sqrt{15} - \sqrt{3})$

b) Let $\sin^{-1}\frac{1}{3} = D$ $\quad \cos^{-1}\frac{1}{2} = B$

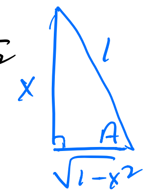
$\cos(D-B) = \cos D\cos B + \sin B\sin D$

$= \frac{\sqrt{8}}{3}\times\frac{1}{2} + \frac{\sqrt{3}}{2}\times\frac{1}{3}$

$\therefore\ A = \frac{1}{6}(\sqrt{8} + \sqrt{3})$

Q8 Let $\cos^{-1}\sqrt{1-x^2} = A$

$\therefore \cos A = \sqrt{1-x^2}$



$\therefore \sin A = x$

$\Rightarrow A = \sin^{-1} x$

$\therefore \sin^{-1} x = \cos^{-1}\sqrt{1-x^2}$

true if $0 \le x \le 1$

Q9 Let $\tan^{-1} A = x \quad \tan^{-1} B = y$

$\therefore A = \tan x \quad B = \tan y$

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

$$= \frac{A - B}{1 + AB}$$

$$\therefore x - y = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$$

$$= \tan^{-1} x - \tan^{-1} y$$

Q11 a) $f(x) = 1 + 2\sin 3x$

range $-1 \le y \le 3$

b) $y = 1 + 2\sin 3x$

$x = 1 + 2\sin 3y$

$3y = \sin^{-1}\left(\frac{x-1}{2}\right)$

$y = \frac{1}{3}\sin^{-1}\left(\frac{x-1}{2}\right)$

c) $D: -1 \le x \le 3$ from range of $f(x)$

or $-1 \le \frac{x-1}{2} \le 1$

$-2 \le x - 1 \le 2$

$-1 \le x \le 3$

$R: -\frac{\pi}{6} \le y \le \frac{\pi}{6}$

Q16 Now $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\therefore \left|\frac{a-p}{1+ap}\right| = \left|\frac{r-a}{1+ar}\right|$ angles are acute

$\therefore \frac{a-p}{1+ap} = \frac{r-a}{1+ar}$

$\therefore (a-p)(1+ar) = (r-a)(1+ap)$

$(a-p)(1+ar) + (a-r)(1+ap) = 0$

Q17 Let $\tan\alpha = \frac{35}{x}$

and $\tan(\alpha + \theta) = \frac{40}{x}$

$$\frac{40}{x} = \frac{\frac{35}{x} + \tan\theta}{1 - \frac{35}{x}\tan\theta}$$

$$\frac{40}{x} = \frac{35 + x\tan\theta}{x - 35\tan\theta}$$

$\tan\theta(x^2 + 1400) = 40x - 35x$

$\tan\theta = \frac{5x}{x^2 + 1400}$

$\theta = \tan^{-1}\left(\frac{5x}{x^2+1400}\right)$