5A

Q1

Q2

Q3

Q4

Q5 a) $\sin 60^\circ = \frac{2}{x}$

$x = 2 \cdot 3$

b) $\tan 20^\circ = \frac{x}{7}$

$x = 2 \cdot 5$

c) $\cos 40^\circ = \frac{x}{x^2}$

$x = \frac{1}{\cos 40^\circ}$

$= 1 \cdot 3$

Q6 a) $x^2 = 25^2 - 15^2$

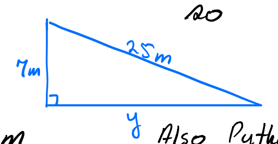
$x = 20$

b) $y^2 = 25^2 - 7^2$

$y = 24$

Slip away 4m.

Note: Pythag. triad 3, 4, 5
so 6, 8, 10
15, 20, 25 etc.



Also Pythag. triad 7, 24, 25

Q7 a) $\sin 60^\circ = \frac{x}{8}$

$x = 6 \cdot 93\text{m}.$

b) $\cos 60^\circ = \frac{y}{8}$

$y = 4\text{m}.$

Q8

Q9

Q10 $\tan 5^\circ = \frac{120}{x}$

$x = 1371 \cdot 6\text{m}$

Q11 a) $\tan 40^\circ = \frac{h}{25}$

$h = 20 \cdot 977$

$\doteqdot 21\text{ m}.$

b) $\tan 55^\circ = \frac{h}{x}$

$x = \frac{20 \cdot 977}{\tan 55^\circ}$

$= 14 \cdot 69$

$\doteqdot 15\text{m}.$

Q12 $\tan 60^\circ = \frac{120}{x}$ $\quad \tan 45^\circ = \frac{120}{y}$

$PQ = x + y$

$= 69 \cdot 28 + 120$

$= 189 \cdot 3\text{ m}.$

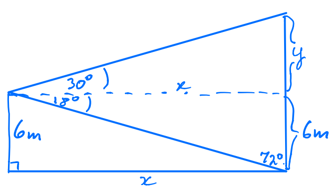
Q13

a) $\tan 72^\circ = \frac{x}{6}$

$x = 18 \cdot 47\text{ m}$

b) $\tan 30^\circ = \frac{y}{x}$

$y = 10 \cdot 66$

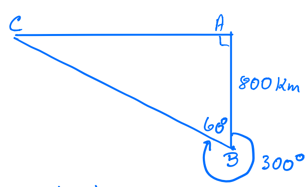


Q14

Q15

Q16 $\cos 60^\circ = \frac{800}{BC}$

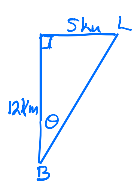
$BC = 1600\text{m}.$



Q17 $\tan \theta = \frac{5}{12}$

$\theta = 22^\circ 37'$

bearing $23^\circ T$



Q18 a) $\cos 60^\circ = \frac{x}{12}$

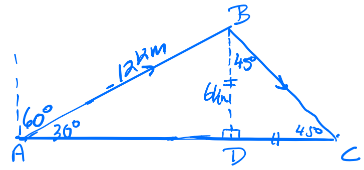
$x = 6$ km.

b) $\cos 30^\circ = \frac{AD}{12}$

$AC = AD + DC$

$= 10.4 + 6$

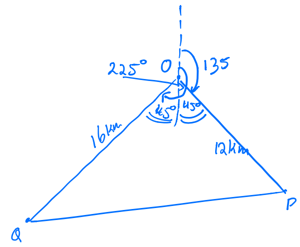
$= 16.4$ km



Q19

$\angle QOP = 90^\circ$

$\therefore QP = 20$ km



P1 a) 90°

b) 500km

c) $\tan\theta = \frac{3}{4}$

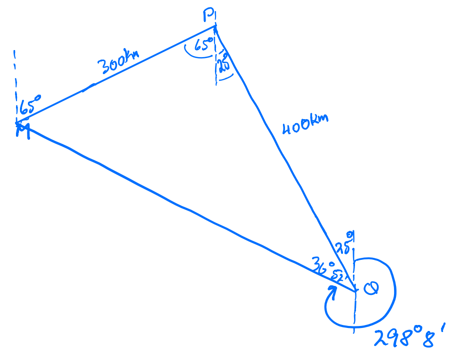
$\theta = 36^\circ 52'$

$\angle PQM = 36^\circ 52'$

d) $\angle PMQ = 53^\circ\, 8'$

bearing $118^\circ 8'$

and $298^\circ 8'$



P2 a) In Δ AED $\tan\alpha = \frac{DE}{AE}$

$\therefore DE = AE \tan\alpha$ now since $AE = BC$

$= BC \tan\alpha$

b) In ΔABC, $\angle ACB = \beta$ (alt $\angle$s)

$\therefore \tan\beta = \frac{h}{BC} \Rightarrow BC = \frac{h}{\tan\beta}$

$DE = \frac{h\tan\alpha}{\tan\beta}$

c) $CD = h - DE$

$= h - \frac{h\tan\alpha}{\tan\beta}$

$= h\left[1 - \frac{\tan\alpha}{\tan\beta}\right] = h\left[\frac{\tan\beta - \tan\alpha}{\tan\beta}\right]$

5B

Q1

Q2

Q3 a) $LHS = \frac{1+\frac{1}{\sqrt3}}{1-\frac{1}{\sqrt3}} \times \frac{\sqrt3}{\sqrt3}$

$= \frac{\sqrt3+1}{\sqrt3-1} \times \frac{\sqrt3+1}{\sqrt3+1}$

$= \frac{4+2\sqrt3}{2} = 2+\sqrt3 = RHS.$

b) $LHS = \frac{\sqrt3}{2} \times \frac{1}{\sqrt2} - \frac{1}{2} \times \frac{1}{\sqrt2}$

$= \frac{\sqrt3-1}{2\sqrt2} \times \frac{\sqrt2}{\sqrt2}$

$= \frac{\sqrt6-\sqrt2}{4} = RHS.$

c) $LHS = \frac{\sqrt3-1}{1+\sqrt3} \times \frac{\sqrt3-1}{\sqrt3-1}$

$= \frac{4-2\sqrt3}{2} = 2-\sqrt3$

$= RHS.$

d) $LHS = \frac{1}{\frac{2}{\sqrt3} - \frac{1}{\sqrt3}} = \frac{1}{\frac{1}{\sqrt3}}$

$= \sqrt3 = RHS.$

Q4

Q5

Q6 a) $\tan 30^\circ = \frac{x}{6}$

$x = 6 \times \frac{1}{\sqrt3} \times \frac{\sqrt3}{\sqrt3}$

$= 2\sqrt3$

b) $x = 2\sqrt3$ $\quad x^2 = 16 - 4$

c) $x = 9 - 9 \times \frac{1}{\sqrt3}$ $\quad \tan 30^\circ = \frac{y}{9}$

$= 9 - 3\sqrt3$

d) $\tan 30^\circ = \frac{20}{20+x} = \frac{1}{\sqrt3}$

$20 + x = 20\sqrt3$

$x = 20\sqrt3 - 20$

Q7 a) exterior angle of $\triangle PQS$ equals sum of interior opposites

$\therefore \angle PSQ = 15^\circ$

$\therefore \triangle PQS$ is isosceles

$\therefore PQ = QS.$

b) $\sin 30^\circ = \frac{1}{2} = \frac{SR}{SQ}$

$\frac{1}{2} = \frac{1}{SQ} \Rightarrow SQ = 2$

$\Rightarrow PQ = 2$

$\tan 30^\circ = \frac{1}{\sqrt3} = \frac{1}{QR} \Rightarrow QR = \sqrt3$

c) $\tan 15^\circ = \frac{SR}{PR} = \frac{1}{2+\sqrt3} \times \frac{2-\sqrt3}{2-\sqrt3}$

$= 2-\sqrt3$

P1 a) $\theta = 15^\circ$ $\quad \tan 2\theta = \tan 30^\circ = \frac{1}{\sqrt3}.$

$\therefore$ substituting in identity

$\frac{2t}{1-t^2} = \frac{1}{\sqrt3}$

b) $1 - t^2 = 2\sqrt3 t$

$t^2 + 2\sqrt3 t - 1 = 0$

c) $t = \frac{-2\sqrt3 \pm \sqrt{12+4}}{2}$

$= -\sqrt3 + 2$ $\quad t > 0$ as $\tan 15^\circ > 0$

d) Use $\tan 45^\circ = \frac{2\tan 22\frac{1}{2}^\circ}{1-\tan^2 22\frac{1}{2}^\circ}$

$1 = \frac{2t}{1-t^2}$

$$1 - t^2 = 2t$$
$$t^2 + 2t - 1 = 0$$
$$t = \frac{-2 \pm \sqrt{8}}{2} \qquad t > 0 \text{ as } \tan 22\tfrac{1}{2}^\circ > 0$$
$$= \sqrt{2} - 1$$

P2 a) $\cos 30^\circ = \frac{\sqrt{3}}{2} = 1 - 2\sin^2 15^\circ$

$$\frac{\sqrt{3}}{2} = 1 - 2s^2$$
$$2s^2 = \frac{2 - \sqrt{3}}{2}$$
$$s = \sqrt{\frac{2 - \sqrt{3}}{2}}$$

b) $\frac{(\sqrt{3} - 1)^2}{2} = \frac{3 - 2\sqrt{3} + 1}{2}$

$$= 2 - \sqrt{3}$$

c) Using a) and b) $2s^2 = \frac{2 - \sqrt{3}}{2} = \frac{(\sqrt{3} - 1)^2}{4}$

$$s^2 = \frac{(\sqrt{3} - 1)^2}{8}$$
$$s = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

5C

Q1 – Q11 see answers.

Q12

a) $\sin 480° = \sin 120°$
$= \sin 60°$
$= \frac{\sqrt{3}}{2}$

b) $\tan 405° = \tan 45°$
$= 1$

c) $\sin 750° = \sin 30°$
$= \frac{1}{2}$

d) $\tan 420° = \tan 60°$
$= \sqrt{3}$

e) $\cos 420° = \cos 60°$
$= \frac{1}{2}$

f) $\sin 570° = \sin 210°$
$= -\sin 30°$
$= -\frac{1}{2}$

Q13

Q14 a) $LHS = \sin 240°$
$= -\sin 60°$
$= -\frac{\sqrt{3}}{2}$

$RHS = 2\sin 120° \cos 120°$
$= 2 \times \frac{\sqrt{3}}{2} \times -\frac{1}{2}$
$= -\frac{\sqrt{3}}{2}$

b) $LHS = \cos 300°$
$= \cos 60°$
$= \frac{1}{2}$

$RHS = \cos^2 150° - \sin^2 150°$
$= \left(-\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2$
$= \frac{1}{2}$

c) $LHS = \sin(210° + 330°)$
$= \sin 540°$
$= \sin 180$
$= 0$

$RHS = \sin 210° \cos 330° + \cos 210° \sin 330°$
$= \left(-\frac{1}{2}\right)\frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{1}{2}\right)$
$= 0$

Q15 a) $\frac{-\sin B}{\cos B} = -\tan B$

b) $\frac{-\cos B}{\cos B} = -1$

c) $\sin B \sin B + \cos B \cos B = 1$

d) $-\sin B \sin B = -\sin^2 B$

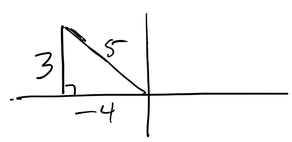
e) $\frac{+\tan B}{-\sec B} = -\sin B$

f) $\frac{\cos B}{\cot B} = \sin B$

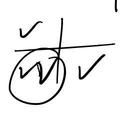
Q16 $LHS = \frac{\sin\theta \sin\theta}{-\cos\theta \cos\theta} = -\tan^2\theta$
$= RHS.$

Q17

P1 a) $\cos \alpha = -\frac{4}{5}$
$\tan \alpha = -\frac{3}{4}$



b) $\tan \alpha = \frac{3}{4}$



c)

$\cot\alpha = -\frac{12}{5}$

P2.

$$\begin{aligned} LHS &= \sin(300^\circ + 210^\circ) \\ &= \sin(510^\circ) = \sin 150^\circ \\ &= \sin 30^\circ \\ &= \frac{1}{2} \\ RHS &= \sin 300^\circ \cos 210^\circ + \cos 300^\circ \sin 210^\circ \\ &= -\frac{\sqrt{3}}{2} \times -\frac{\sqrt{3}}{2} + \frac{1}{2} \times -\frac{1}{2} \\ &= \frac{1}{2} = LHS. \end{aligned}$$

5D

Q 3

a) $\sin^2\theta \times \frac{1}{\sin^2\theta} = 1$

b) $\cot^2\theta - (1+\cot^2\theta) = -1$

c) $\tan^2\theta - \sec^2\theta = \tan^2\theta - (1+\tan^2\theta)$
$= -1$

d) $\sin^2\theta(1-\cos^2\theta) = \sin^2\theta \times \sin^2\theta$
$= \sin^4\theta$

e) $(\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$
$= \sin^2\theta - \cos^2\theta$

f) $\sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$
$= 1 + 2\sin\theta\cos\theta$

g) $1 - \tan^2\theta + 1 + \tan^2\theta = 2$

h) $\frac{\sin^2\theta}{\cos^2\theta} \times \cos^2\theta + \frac{\cos^2\theta}{\sin^2\theta} \times \sin^2\theta$
$= \sin^2\theta + \cos^2\theta = 1$

Q 4 a) $LHS = \tan\theta + \frac{1}{\tan\theta}$
$= \frac{\tan^2\theta + 1}{\tan\theta}$
$= \sec^2\theta \times \frac{\cos\theta}{\sin\theta}$
$= \frac{\sec\theta(\sec\theta \times \cos\theta)}{\sin\theta}$
$= \sec\theta \operatorname{cosec}\theta \times 1$
$= RHS.$

or $LHS = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$
$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$
$= \sec\theta \operatorname{cosec}\theta$
$= RHS.$

b) $LHS = \tan\theta - \tan\theta\cot^2\theta + \cot\theta - \cot\theta\tan^2\theta$
$= \tan\theta - \cot\theta + \cot\theta - \tan\theta$
$= 0 = RHS.$

There are several ways of proving identities. One method is given for these questions

c) $LHS = 2(1 - \sin^2\theta) - 1$
$= 2 - 2\sin^2\theta - 1$
$= 1 - 2\sin^2\theta$
$= RHS.$

d) $LHS = \frac{\sin\theta}{\cos\theta} \times \sin\theta + \cos\theta$
$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta}$
$= \frac{1}{\cos\theta}$
$= \sec\theta = RHS.$

e) $LHS = \frac{1}{\sin\theta\cos\theta} - \frac{\sin\theta}{\cos\theta}$
$= \frac{1 - \sin^2\theta}{\sin\theta\cos\theta}$
$= \frac{\cos^2\theta}{\sin\theta\cos\theta}$
$= \frac{\cos\theta}{\sin\theta}$
$= \cot\theta$
$= RHS.$

f) LHS $= \dfrac{1}{1+\sin^2\theta} + \dfrac{1}{1+\dfrac{1}{\sin^2\theta}}$

$= \dfrac{1}{1+\sin^2\theta} + \dfrac{\sin^2\theta}{\sin^2\theta+1}$

$= \dfrac{1+\sin^2\theta}{1+\sin^2\theta}$

$= 1$

$=$ RHS.

g) LHS $= \dfrac{1+\cos\theta}{1-\cos^2\theta}$

$= \dfrac{1+\cos\theta}{(1+\cos\theta)(1-\cos\theta)}$

$= \dfrac{1}{1-\cos\theta}$

$=$ RHS.

h) LHS $= \dfrac{1-\cos\theta}{(1-\cos\theta)(1+\cos\theta)}$

$= \dfrac{1}{1+\cos\theta}$

$=$ RHS.

i) LHS $= (\sec^2\theta - \tan^2\theta)(\sec^2\theta + \tan^2\theta)$

$= 1 \times (\sec^2\theta + \sec^2\theta - 1)$

$= 2\sec^2\theta - 1$

$=$ RHS.

j) LHS $= \dfrac{\operatorname{cosec}\theta + \sec\theta}{1+\tan\theta} \times \dfrac{\cos\theta}{\cos\theta}$

$= \dfrac{\operatorname{cosec}\theta\cos\theta + 1}{\cos\theta + \sin\theta}$

$= \dfrac{\operatorname{cosec}\theta(\cos\theta + \sin\theta)}{\cos\theta + \sin\theta}$

$= \operatorname{cosec}\theta$

$=$ RHS.

k) LHS $= \dfrac{(1-\sin\theta)(1+\sin\theta)}{1+\sin\theta} + \dfrac{(1-\sin\theta)(1+\sin\theta)}{1-\sin\theta}$

$= 1 - \sin\theta + 1 + \sin\theta$ using $\cos^2\theta = 1 - \sin^2\theta$

$= 2$

$=$ RHS.

l) LHS $= \dfrac{1-\sin\theta + 1 + \sin\theta}{(1+\sin\theta)(1-\sin\theta)}$

$= \dfrac{2}{1-\sin^2\theta}$

$= \dfrac{2}{\cos^2\theta}$

$= 2\sec^2\theta$

$=$ RHS

m) LHS $= \dfrac{\cot^2\theta - 1}{\operatorname{cosec}^2\theta}$

$= \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$=$ RHS

n) LHS $= \left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)^2$

$= \frac{(1+\sin\theta)^2}{\cos^2\theta}$

$= \frac{(1+\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{1+\sin\theta}{1-\sin\theta}$

$=$ RHS.

o) LHS $= (\sec^2\theta - 1)\cos\theta$

$= \sec\theta - \cos\theta$

$=$ RHS

p) LHS $= \frac{\cos^2\theta + (1+\sin\theta)^2}{(1+\sin\theta)\cos\theta}$

$= \frac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{(1+\sin\theta)\cos\theta}$

$= \frac{2(1+\sin\theta)}{(1+\sin\theta)\cos\theta}$

$= 2\sec\theta$

$=$ RHS.

P1

Method 1.

LHS $= x^2 + y^2$

$= a^2\cos^2\theta + a^2\sin^2\theta$

$= a^2(\cos^2\theta + \sin^2\theta)$

$= a^2 =$ RHS.

Method 2.

$\cos\theta = \frac{x}{a}$ $\quad \sin\theta = \frac{y}{a}$

now $\sin^2\theta + \cos^2\theta = 1$

Hence $\frac{y^2}{a^2} + \frac{x^2}{a^2} = 1$

$\therefore x^2 + y^2 = a^2$

P2 a) Using $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$1 + \left(\frac{y}{2}\right)^2 = x^2$

$4x^2 - y^2 = 4$

b) Using $1 + \tan^2\theta = \sec^2\theta$

$1 + \left(\frac{y}{3}\right)^2 = \left(\frac{x}{3}\right)^2$

$\therefore x^2 - y^2 = 9$.

P3

LHS $= x^2 - xy + 1$

$= (\sec\theta + \tan\theta)^2 - (\sec\theta + \tan\theta)2\sec\theta + 1$

$= \sec^2\theta + 2\sec\theta\tan\theta + \tan^2\theta - 2\sec^2\theta - 2\sec\theta\tan\theta + 1$

$= \tan^2\theta - \sec^2\theta + 1$

$= 0 =$ RHS.

5E

Q1

Q2

Q3

Q4

Q5

Q6 a) $\sin\theta = \pm 1$ $\quad\theta = 90°, 270°$

b) $\cos\theta = \pm\frac{\sqrt{3}}{2}$ $\quad\theta = 30°, 150°, 210°, 330°$

c) $\tan\theta = \pm\sqrt{3}$ $\quad\theta = 60°, 120°, 240°, 300°$

d) $\cos\theta = \pm\frac{1}{\sqrt{2}}$ $\quad\theta = 45°, 135°, 225°, 315°$

e) $\tan\theta = \pm\frac{1}{\sqrt{3}}$ $\quad\theta = 30°, 150°, 210°, 330°$

Q7 a) $\sin\theta = \frac{1}{2}$ or $\sin\theta = -1$

$\theta = 30°, 150°$ or $\theta = 270°$

b) $(2\sin\theta - 1)(\sin\theta - 1) = 0$

$\sin\theta = \frac{1}{2}$ or $\sin\theta = 1$

$\theta = 30°, 150°, 90°$

c) $(2\cos\theta - 3)(\cos\theta + 1) = 0$

$\cos\theta = \frac{3}{2}$ or $\cos\theta = -1$

no soln $\quad\theta = 180°$

d) $(\sec\theta + 2)(\sec\theta - 1) = 0$

$\cos\theta = -\frac{1}{2}$ $\quad\cos\theta = 1$

$\theta = 120°, 240°, 0°, 360°$

Q8

Q9 a) $2\theta = 45°, 225°, 405°, 585°$ $\quad 2\theta \in [0°, 720°]$

$\theta = 22\frac{1}{2}°, 112\frac{1}{2}°, 202\frac{1}{2}°, 292\frac{1}{2}°$

b) $2\theta = 120°, 240°, 480°, 600°$ $\quad 2\theta \in [0°, 720°]$

$\theta = 60°, 120°, 240°, 300°$

c) no soln. $\quad\frac{\theta}{2} \in [0, 180°]$

d) $3\theta = 45°, 315°, 405°, 675°, 765°, 1035°$ $\quad 3\theta \in [0°, 1080°]$

$\theta = 15°, 105°, 135°, 225°, 255°, 345°$

Q10 a) $\theta + 60° = 330°, 390°$ $\quad\theta + 60° \in [60°, 420°]$

$\theta = 270°, 330°$

b) $\theta - 45° = 30°, 210°$ $\quad\theta - 45° \in [-45°, 315°]$

$\theta = 75°, 255°$

c) $\theta + 150° = 210°, 330°$ $\quad\theta + 150° \in [150°, 510°]$

$\theta = 60°, 180°$

d) $\theta + 210° = 300°, 480°$ $\quad\theta + 210° \in [210°, 570°]$

$\theta = 90°, 270°$

P1 a) $\sin(2\theta - 60) = 1$ $\quad 2\theta \in [0°, 720°]$

$2\theta - 60° = 90°, 450°$ $\quad 2\theta - 60° \in [-60°, 660°]$

$2\theta = 150°, 510°$

$\theta = 75°, 255°$

b) $\cos(3\theta + 45°) = -\frac{1}{\sqrt{2}}$ $\quad 3\theta + 45° \in [45°, 1125°]$

$3\theta + 45° = 135°, 225°, 495°, 585°, 855°, 945°$

$\theta = 30°, 60°, 150°, 180°, 270°, 300°$

P2 a)

$\cos^2\theta = \sin\theta - 1$

$1 - \sin^2\theta = \sin\theta - 1$

$\sin^2\theta + \sin\theta - 2 = 0$

$(\sin\theta - 1)(\sin\theta + 2) = 0$

$\sin\theta = 1$ or $\sin\theta = -2$

$\theta = 90^\circ$ no soln.

b)

$\sec^2\theta - \tan\theta - 1 = 0$

$1 + \tan^2\theta - \tan\theta - 1 = 0$

$\tan\theta(\tan\theta - 1) = 0$

$\tan\theta = 0$ $\tan\theta = 1$

$\theta = 0^\circ, 180^\circ, 360^\circ, 45^\circ, 225^\circ$

c)

$\cos^2\theta - \sin^2\theta + 3\cos\theta + 2 = 0$

$2\cos^2\theta + 3\cos\theta + 1 = 0$

$(2\cos\theta + 1)(\cos\theta + 1) = 0$

$\cos\theta = -\frac{1}{2}$ or $\cos\theta = -1$

$\theta = 120^\circ, 240^\circ, 180^\circ$

d)

$\tan^2\theta - \sec\theta - 1 = 0$

$\sec^2\theta - \sec\theta - 2 = 0$

$(\sec\theta - 2)(\sec\theta + 1) = 0$

$\sec\theta = 2$ or $\sec\theta = -1$

$\theta = 60^\circ, 300^\circ, 180^\circ$

P3

$4\cos^3\theta = \cos 3\theta + 3\cos\theta$

$8\cos^3\theta = 2\cos 3\theta + 6\cos\theta = 6\cos\theta - 1$

$\therefore 2\cos 3\theta = -1$

$3\theta = 120^\circ, 240^\circ, 480^\circ$

$\theta = 40^\circ, 80^\circ, 160^\circ$

Note: $3\theta \in [0^\circ, 540^\circ]$

P4

squaring introduces extra solutions.

5F

Q1 a) $\frac{x}{\sin 30^\circ} = \frac{10}{\sin 45^\circ}$

$x = 10\sqrt{2} \times \frac{1}{2} = 5\sqrt{2}$

b) $\beta = 50^\circ$ (angle sum of $\triangle = 180^\circ$)

$\frac{x}{\sin 50^\circ} = \frac{8}{\sin 60}$

$x = 7.08$

Q2 a) $\angle BAP = 15^\circ$

$\therefore \frac{AP}{\sin 30^\circ} = \frac{30}{\sin 15^\circ} \Rightarrow AP = \frac{30 \sin 30^\circ}{\sin 15^\circ}$

b) In $\triangle PAC$ $AP = x\sqrt{2}$

$x = \frac{30}{\sqrt{2}} \times \frac{1}{2 \sin 15^\circ} = 41\text{m}$

Q3 a) $\sin 30^\circ = \frac{12}{AE}$

$AE = 24\text{m}$

b) In $\triangle AED$, $\angle AED = 120^\circ$

so $\angle EDA = 20^\circ$

$\therefore \frac{ED}{\sin 40^\circ} = \frac{24}{\sin 20^\circ}$

$ED = 45\text{m}.$

Q4 a) $\angle OAT = 17^\circ$

$\sin 17^\circ = \frac{30}{AT}$

$AT = 102.61\text{ m}$

b) $\angle ATB = 5^\circ$, $\angle TAB = 73^\circ$ $\angle TBA = 102^\circ$

c) $\frac{AB}{\sin 5^\circ} = \frac{102.61}{\sin 102^\circ}$

$AB = 9.14\text{m}.$

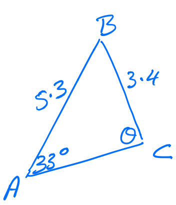
Q5 a) $\frac{\sin\theta}{5} = \frac{\sin 60^\circ}{7}$

$\theta = 38^\circ 13'$ or $180^\circ - 38^\circ 13' = 141^\circ 47'$

b) $\frac{\sin\theta}{6} = \frac{\sin 30^\circ}{10}$

$\theta = 17^\circ 27'$, $162^\circ 33'$

Q6 $\frac{\sin\theta}{5.3} = \frac{\sin 33^\circ}{3.4}$



$\theta = 58^\circ 6'$, $121^\circ 54'$

Case 1 If $\theta = \angle BCA = 58^\circ 6'$

then $\angle ABC = 88^\circ 54'$ and

$\frac{AC}{\sin 88^\circ 54'} = \frac{3.4}{\sin 33^\circ}$

$AC = 6.2\text{cm}.$

Case 2 If $\angle BCA = 121^\circ 54'$

then $\angle ABC = 25^\circ 6'$ and.

$\frac{AC}{\sin 25^\circ 6'} = \frac{3.4}{\sin 33^\circ}$

$AC = 2.6\text{cm}.$

Q7 a) $Area = \frac{1}{2}ab\sin C = \frac{1}{2} \times 10 \times 10 \sin 60$

$= 50\frac{\sqrt{3}}{2} = 25\sqrt{3}\ cm^2$

b) $9\sqrt{3} = \frac{1}{2}a^2\frac{\sqrt{3}}{2}$

$a^2 = 36$

$a = 6$

Q8 $AC = 8\ cm$ isosceles Δ

$\angle BAC = 80^\circ$

$Area = \frac{1}{2} \times 8 \times 8 \sin 80^\circ$

$= 31.51\ cm^2$

Q9 $Area = \frac{1}{2}ab\sin\theta$

$14 = \frac{1}{2} \times 4 \times 8 \times \sin\theta$

$\theta = 61^\circ 3'$ or $118^\circ 57'$

P1 a) $Area\ \Delta ABC = \frac{1}{2} \times 12 \times 20 \sin 60^\circ$ (using $\frac{1}{2}ab\sin C$ = Area)

$= 120 \times \frac{\sqrt{3}}{2}$

$= 60\sqrt{3}\ u^2$

b) $60\sqrt{3} = \frac{1}{2} \times 12x \sin 45 + \frac{1}{2} \times 20x \sin 15^\circ$

$= x(6.83)$

$\therefore x = 15.2\ u.$

P2 a) $\sin B = \frac{h}{AD} \Rightarrow AD = \frac{h}{\sin B}$

b) $\angle BDA = \alpha - B \quad \angle ABD = 180^\circ - \alpha.$

c) In ΔABD

$\frac{\sin(\alpha - B)}{x} = \frac{\sin(180 - \alpha)}{AD}$

$\frac{\sin(\alpha - B)}{x} = \frac{\sin\alpha \sin B}{h}$

$\therefore h = \frac{x\sin\alpha\sin B}{\sin(\alpha - B)}$

P3 a) Area of $\Delta ABC = \frac{1}{2}ab\sin C$

b) b

c) using sine rule with angles A and B

so $\frac{b}{\sin B} = \frac{a}{\sin A}$

$b = \frac{a\sin B}{\sin A}$

d) Area of $\Delta ABC = \frac{1}{2}a \times \frac{a\sin B}{\sin A}\sin C$

$= \frac{a^2\sin B\sin C}{2\sin A}$

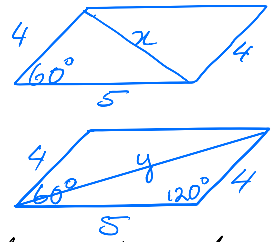
5G

Q1 $x^2 = 4^2 + 5^2 - 2\times4\times5\cos 60°$
$= 21$

$x = 4.58\text{cm}.$

$y^2 = 4^2 + 5^2 - 2\times4\times5\cos 120°$
$= 61$

$y = 7.81\text{ cm}$



Q2 a) Largest angle is opposite largest side

$\cos\theta = \dfrac{5^2+3^2-7^2}{2\times5\times3}$

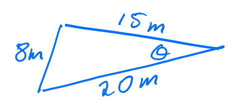
$= -\frac{1}{2}$

$\theta = 120°$

b) Area of $\Delta = \frac{1}{2}\times5\times3\sin 120°$
$= \dfrac{5\sqrt{3}}{4}\text{ cm}^2$

Q3

$\cos\theta = \dfrac{15^2+20^2-8^2}{2\times15\times20}$



$\theta = 20°46'$

Q4 $AB^2 = 8^2+8^2-2\times8\times8\cos 60°$

$AB = 8\text{ cm}$

A result we expected as it is an equilateral triangle – all angles 60°

Q5

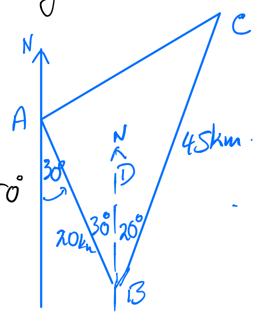
a) alternate angles are equal

$\angle ABD = 30°$ $\angle DBC = 20°$

$\therefore \angle ABC = 50°$

b) $AC^2 = 20^2+45^2-2\times20\times45\cos 50°$

$AC = 36\text{km}.$



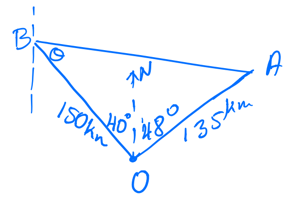
Q6 a) car A 135km

car B 150km

b) $AB^2 = 135^2+150^2-2\times135\times150\cos 88°$

$AB = 198.27\text{ km}$

c) $\dfrac{\sin\theta}{135} = \dfrac{\sin 88}{198.27}$



$\sin\theta = 0.68047$

$\theta = 42°53'$ angle acute

bearing $180 - (42°53' + 40°)$

$= 97°7'T$

Q7 a) $q^2 = x^2 + 4^2 - 2\times x\times4\cos 60°$

$= x^2 - 4x + 16$ Note $\cos 60° = \frac{1}{2}$

b) axis of symmetry $x = -\dfrac{b}{2a} = \dfrac{4}{2} = 2$

when $x = 2$ $y = 12$ $\therefore$ Min is 12

c) $q = \sqrt{12}$

$= 2\sqrt{3}$

d) ΔPQR in this case has sides

$2,\ 2\sqrt{3},\ 4$ and since

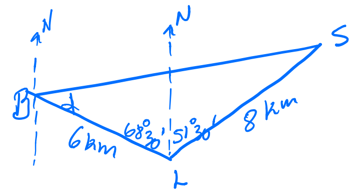
$(2\sqrt{3})^2 + 2^2 = 4$ it is right angled.

e) $30°$

Q8

a)

b) $68°30' + 51°30' = 120°$



$SB^2 = 6^2 + 8^2 - 2 \times 6 \times 8 \times \cos 120°$

$= 100 - 96(-\frac{1}{2})$

$= 148$

$SB = 2\sqrt{37}$ km

c) $\frac{\sin \alpha}{8} = \frac{\sin 120}{2\sqrt{37}}$

$\sin \alpha = 0.56949$

$\alpha = 34° 43'$

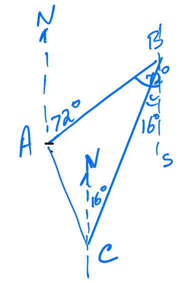
required angle $= 180 - (34°43' + 68°30')$

$= 76° 47'$

bearing $77°$

Q9

a) see answers



b) $\angle ABS = 72°$

$\angle CBS = 16°$ using bearing 196°

$\therefore \angle ABC = 56°$

c) $AC^2 = 152^2 + 240^2 - 2 \times 150 \times 240 \cos 56°$

$= 39905.29$

$AC = 199.76$

$\doteqdot 200$ km.

d) $\cos \angle ACB = \frac{199.76^2 + 240^2 - 152^2}{2 \times 199.76 \times 240}$

$= 0.7759$

$\angle ACB = 39°$

$\therefore \angle NCA = 23°$

$\therefore$ bearing $360° - 23° = 337°$

Q10

$(6\sqrt{3})^2 = (x-3)^2 + (x+3)^2 - 2 \times (x-3)(x+3) \cos 60°$

$108 = x^2 - 6x + 9 + x^2 + 6x + 9 - x^2 + 9$

$x^2 = 81$

$x = 9$

P1

a) $QS^2 = 4^2 + 2^2 - 2 \times 4 \times 2 \times \cos 60° = 12$

$QS^2 = (4-x)^2 + (2-x)^2$

$\therefore 2x^2 - 12x + 20 = 12$

$x^2 - 6x + 4 = 0$

b) $x = \frac{6 \pm \sqrt{36-16}}{2}$

$= 3 - \sqrt{5}$ Note $2 - x > 0$

P2

You get Pythagoras' Theorem!
This is not a surprise since $C = 90°$
so we have a right-angled triangle

P3

a) $c^2 = x^2 + x^2 - 2x^2 \cos\theta$

$= 2x^2(1 - \cos\theta)$

b) If $c = \frac{x}{2}$ $\frac{x^2}{4} = 2x^2(1 - \cos\theta)$

$\therefore \frac{1}{8} = 1 - \cos\theta$

$\cos\theta = \frac{7}{8}$

$\theta = 29°$

c) If $c = x$ $\quad x^2 = 2x^2(1-\cos\theta)$

$\cos\theta = \frac{1}{2}$

$\theta = 60°$ and triangle is equilateral. This was obvious as all three sides are equal.

P4 a) $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

Since $-1 < \cos A < 1$

$\dfrac{b^2+c^2-a^2}{2bc} < 1$

$\therefore b^2+c^2-a^2 < 2bc$

b) $b^2 - 2b + c^2 < a^2$

$(b-c)^2 < a^2$

$b - c < a$

$b < a + c$

c) The sum of two sides of a triangle is larger than the third side

5H

Q1 a) $CF^2 = BF^2 + BC^2$

$= a^2 + a^2 = 2a^2$

$CF = a\sqrt{2}$

b) $CE^2 = EF^2 + CF^2$

$= a^2 + 2a^2 = 3a^2$

$CE = a\sqrt{3}$

c) $\tan \angle ECF = \frac{EF}{FC} = \frac{a}{a\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$

$\angle ECF = 35° 16'$

Q2

$\tan 30° = \frac{100}{BC}$

$BC = 173 \cdot 21\,m$

$\tan 25° = \frac{100}{BD}$

$BD = 214 \cdot 45\,m$

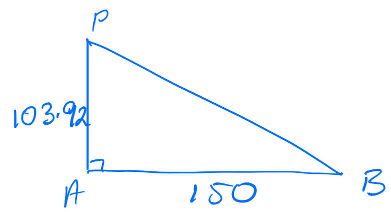
Q3 a) $\tan 30° = \frac{60}{AP}$

$AP = 103 \cdot 92\,m$

b) $\angle PAB = 90°$

$BP^2 = AP^2 + AB^2$

$BP = 182 \cdot 48\,m$



$\tan \angle QBP = \frac{60}{BP}$

$\angle QBP = 18° 12'$

Q4 a) $\sin 30° = \frac{h}{5}$

$h = 2 \cdot 5\,m.$

b) $\sin \angle ADB = \frac{2 \cdot 5}{4}$

$\angle ADB = 38° 41'$

Q5 a) $\angle QPA = 75°$

$\tan 75° = \frac{AQ}{h}$

$\therefore AQ = h \tan 75°$

b) $BQ = h \tan 78°$

c) A is due east of Q and B is south of Q

$\therefore \angle AQB = 90°$

d) $800^2 = h^2 \tan^2 78° + h^2 \tan^2 75°$

$h^2 = \frac{800^2}{\tan^2 78° + \tan^2 75°}$

$= 17747 \cdot 3373$

$h = 133\,m.$

Q6 a) $AC^2 = 10^2 + 10^2$

$AC = 10\sqrt{2}\,m$

b) $EG^2 = EF^2 + FG^2$

$= 64 + 25$

$EG = \sqrt{89}\,m.$

c) $EG \perp DC$

$\tan \angle EDG = \frac{EG}{5} = \frac{\sqrt{89}}{5}$

$\angle EDG = 62^\circ 5'$

d) $\tan \angle EGF = \frac{EF}{FG} = \frac{8}{5}$

$\angle EGF = 58^\circ$

e) $\tan \angle EAF = \frac{EF}{AF} = \frac{8}{5\sqrt{2}}$

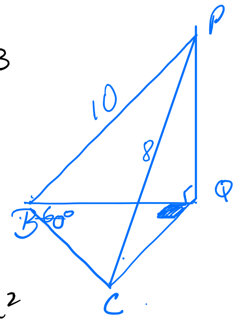
$\angle EAF = 48^\circ 32'$

f) $\angle AEF = 90 - 48^\circ 32'$

$= 41^\circ 28'$

$\angle AEC = 82^\circ 56'$ ($2 \times \angle AEF$)

Q7 a) $\frac{CQ}{BQ} = \tan 60^\circ = \sqrt{3}$



b) $CQ^2 = 64 - h^2$

$BQ^2 = 100 - h^2$

$\therefore \frac{64 - h^2}{100 - h^2} = 3$

c) $64 - h^2 = 300 - 3h^2$

$h^2 = 118$

$h = 10.86$

Q8 a) $\tan 30^\circ = \frac{h}{90}$

$h = 90 \times \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 30\sqrt{3}$

b) Let B be base of pole

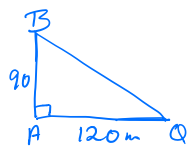
$\angle QAB = 90^\circ$

i) $QB^2 = 120^2 + 90^2$

$QB = 150$ m.

ii) $\tan \angle PQB = \frac{30\sqrt{3}}{150}$

$\angle PQB = 19^\circ 6'$



iii) $\tan \angle BQA = \frac{3}{4}$

$\angle BQA = 36^\circ 52'$

bearing of pole from Q $= 270^\circ + 36^\circ 52'$

$= 306^\circ 52'$

Q9 a)

b) $EB = 6\sqrt{2}$

$OB = 3\sqrt{2}$

$\tan \alpha = \frac{PO}{OB} = \frac{4}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{2\sqrt{2}}{3}$

c) $\tan B = \frac{PO}{OC} = \frac{4}{3}$

Q10 a) $\angle OAB = 90^\circ$

$OA^2 + AB^2 = OB^2$

$\angle OMA = 72^\circ$ and $\angle OMB = 78^\circ$

$\tan 72^\circ = \frac{OA}{h}$ $\tan 78^\circ = \frac{OB}{h}$

$\therefore h^2 \tan^2 72^\circ + 1500^2 = h^2 \tan^2 78^\circ$

$h^2 (\tan^2 78^\circ - \tan^2 72^\circ) = 1500^2$

$h = \frac{1500}{\sqrt{\tan^2 78^\circ - \tan^2 72^\circ}}$

b) $h = 421.55$ m

Q11

a) $\angle ATO = 65°$

$\tan 65° = \frac{AO}{h}$

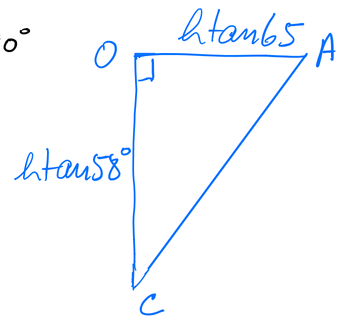
$\therefore AO = h\tan 65°$

b) Similarly

$BO = h\tan 60°$

$CO = h\tan 58°$

c) $\triangle AOC$ is right angled as $\angle AOC = 90°$



d)

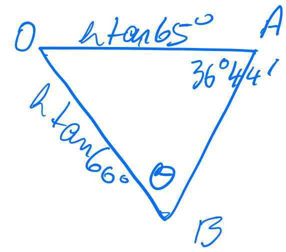
$$\tan \angle OAC = \frac{h\tan 58°}{h\tan 65°}$$

$$= 0.746248$$

$$\angle OAC = 36° 44'$$

e)

$$\frac{\sin\theta}{h\tan 65°} = \frac{\sin 36°44'}{h\tan 60°}$$



$\sin\theta = 0.7405$

$\theta = 47°46'$ or $132°14'$

f) $\angle AOB = 11° 2'$ or $95° 30'$

Since $\angle AOC = 90°$

$\angle AOB \neq 95° 30'$

$\therefore$ bearing of B is $11° 2'$

P1

$OP' = \sqrt{a^2+b^2}$

$OP^2 = (OP')^2 + (PP')^2$

$= a^2+b^2+c^2$

$\therefore OP = \sqrt{a^2+b^2+c^2}$

P2

a) $\cot\alpha = \frac{OA}{h} \Rightarrow OA = h\cot\alpha$

$OB = h\cot\beta$

b) Since $\angle AOB = 90°$

$$d^2 = h^2\cot^2\alpha + h^2\cot^2\beta$$

$$d^2 = \frac{h^2}{\tan^2\alpha} + \frac{h^2}{\tan^2\beta}$$

$$d^2 = h^2\,\frac{\tan^2\beta + \tan^2\alpha}{\tan^2\alpha\tan^2\beta}$$

$$\therefore h^2 = \frac{d^2\tan^2\alpha\tan^2\beta}{\tan^2\alpha + \tan^2\beta}$$

$$\therefore h = \frac{d\tan\alpha\tan\beta}{\sqrt{\tan^2\alpha + \tan^2\beta}}$$

Chapter 5 Review

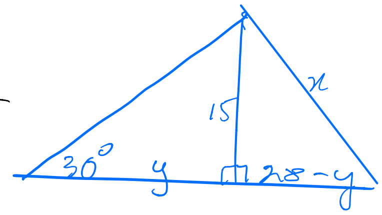
Q1

Q2

$\tan 30^\circ = \frac{15}{y}$

$y = 15\sqrt{3}$

$\therefore x^2 = 15^2 + (28 - 15\sqrt{3})^2$



$x = 15.14$

Q3

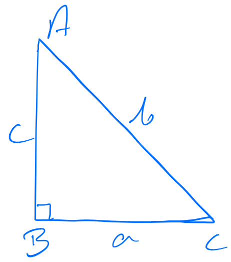
If $\sin A = \cos A$

$\frac{a}{b} = \frac{c}{b}$

$\therefore a = c$

$\therefore \triangle ABC$ is isosceles

$\therefore \angle BAC = \angle BCA = 45^\circ$



or since $\cos A = \sin(90 - A)$

$\sin A = \sin(90^\circ - A)$

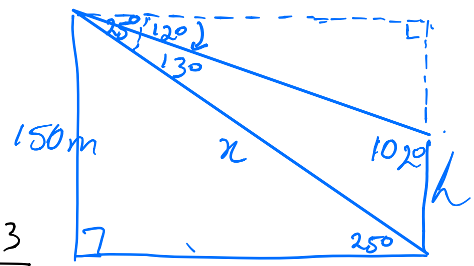
$2A = 90^\circ$

$A = 45^\circ \quad B = 45^\circ$

Q4

$\sin 25^\circ = \frac{150}{x}$

$x = 354.93\,m$



$\frac{h}{\sin 13^\circ} = \frac{354.93}{\sin 102^\circ}$

$h = 81.63\,m$

Q5 a)

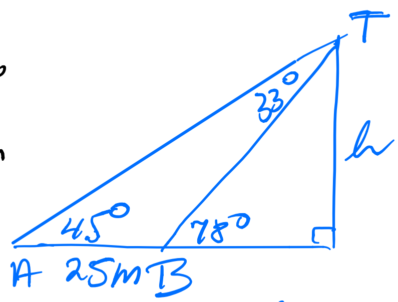
$\frac{TB}{\sin 45^\circ} = \frac{25}{\sin 33^\circ}$

$TB = 32.46\,m$

b)

$\sin 78^\circ = \frac{h}{TB}$

$h = 31.75\,m$

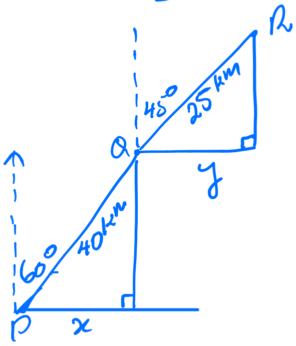


Q6

$x + y$

$= 40\cos 30^\circ + 25\cos 45^\circ$

$= 20\sqrt{3} + \frac{25\sqrt{2}}{2}$ km



Q7

$\angle ADB = 55^\circ$

In $\triangle ADB$ $\sin 55^\circ = \frac{AB}{x}$

$\therefore AB = x \sin 55^\circ$

Q8 a)

$\cos\theta = \sin(90^\circ - \theta) = \sin 30^\circ$

$\therefore 90^\circ - \theta = 30^\circ$

$\theta = 60^\circ$

or $\sin\theta = \cos(90 - \theta)$

$\sin 30^\circ = \cos 60^\circ$

$\therefore \cos\theta = \cos 60^\circ$

$\theta = 60^\circ$

b)

$\sin 2\theta = \cos 42^\circ = \sin 48^\circ = \sin 132^\circ$

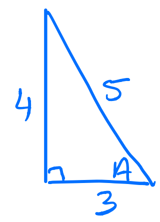
$\therefore \theta = 24^\circ, 66^\circ$

Q9

$$\frac{3\sin A - \cos A}{4\sin A - 5\cos A}$$

$$= \frac{3 \times \frac{4}{5} - \frac{3}{5}}{4 \times \frac{4}{5} - 5 \times \frac{3}{5}} = \frac{12-3}{16-15}$$

$= 9$



Q10 a)

$$\frac{AC}{\sin 120^\circ} = \frac{60}{\sin 30^\circ}$$

$AC = 60 \times \frac{\sqrt{3}}{2} \times 2$

$= 60\sqrt{3}$

Note: $\triangle ADC$ is isosceles $CD = AD = 60$

b)

$CB = 60\sqrt{3}\cos 40^\circ$

$AB = 60\sqrt{3}\sin 40^\circ$

Area of $ABCD = \frac{1}{2} \times 60\sqrt{3}\cos 40^\circ \times 60\sqrt{3}\sin 40^\circ$

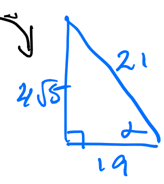
$+ \frac{1}{2} \times 60 \times 60 \sin 120^\circ$

$= 4217.83\ u^2$

Q11

Smallest angle is opposite smallest side

$$\cos\alpha = \frac{9^2 + 7^2 - 4^2}{2 \times 9 \times 7} = \frac{19}{21}$$



Area of $\triangle = \frac{1}{2} ab \sin C$

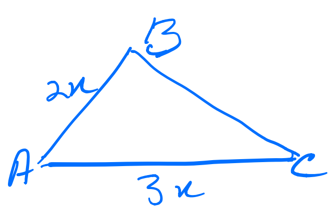
$= \frac{1}{2} \times 9 \times 7 \times \frac{4\sqrt{5}}{21}$

$= 6\sqrt{5}$

Q12

If $\cos A = \frac{3}{5}$ then

$\sin A = \frac{4}{5}$



Area $= 60 = \frac{1}{2} \times 2x \times 3x \times \frac{4}{5}$

$24x^2 = 600$

$x = 5$ cm

Q13

a) $\angle DAB = 50°$

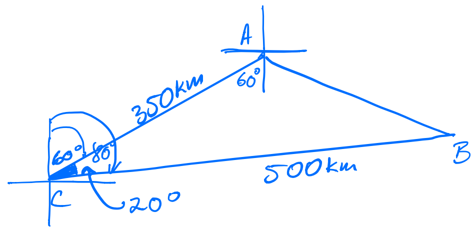
$\angle DBA = 70°$

b) $\dfrac{AB}{\sin 60°} = \dfrac{40}{\sin 70°}$

c) $AB = 37\,cm$

Q14

a)



b) Using cos rule

$AB = \sqrt{350^2 + 500^2 - 2 \times 350 \times 500 \cos 20°}$

Q15

$\tan 34°15' = \dfrac{50}{BC} \rightarrow BC = 50 \cot 34°15'$

$\tan 12°40' = \dfrac{40}{AC} \rightarrow AC = 40 \cot 12°40'$

$AB^2 = AC^2 - BC^2$

$AB = 162 \cdot 12\,m.$

Q16

a) The exterior angle of a triangle is equal to the sum of the interior opposite angles.

b) $\sin 2\alpha = \dfrac{AB}{AD} = \dfrac{AB}{x}$

$\therefore AB = x \sin 2\alpha$

c) $BD = x \cos 2\alpha$

$BC = BD + DC$

$= x \cos 2\alpha + x$

d) $\tan \alpha = \dfrac{AB}{BC} = \dfrac{x \sin 2\alpha}{x \cos 2\alpha + x}$

$= \dfrac{\sin 2\alpha}{1 + \cos 2\alpha}$

Q17

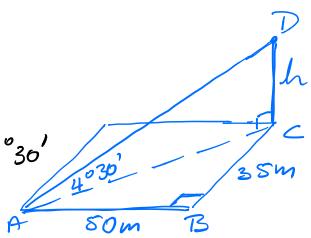
$AC = \sqrt{50^2 + 35^2}$

$= 61 \cdot 0328$

$\tan 4°30' = \dfrac{h}{AC}$

$h = 61 \cdot 0328 \tan 4°30'$

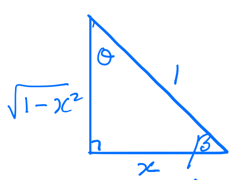
$= 4 \cdot 8\,m$



Q18

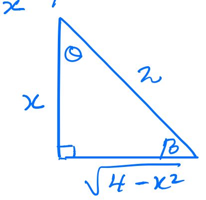
a) $\cos \theta = \sqrt{1 - x^2}$

$\tan \theta = \dfrac{x}{\sqrt{1 - x^2}}$



b)

$\tan \beta = \dfrac{x}{\sqrt{4 - x^2}}$



c) $\dfrac{x}{\sqrt{1 - x^2}} = 3\left(\dfrac{x}{\sqrt{4 - x^2}}\right)$

$\dfrac{x^2}{1 - x^2} = \dfrac{9x^2}{4 - x^2} \qquad x \neq 0$

$9(1 - x^2) = 4 - x^2$

$8x^2 = 5 \implies x = \sqrt{\dfrac{5}{8}}$