2A

Q1 – Q 24

P1

a) Range $[-\frac{2\pi}{3}, \frac{2\pi}{3}]$, period is $\frac{2\pi}{3}$
The more obvious graph is the negative sine curve
$y = -\frac{2\pi}{3}\sin 3x$
Shifting a negative sine curve right $\frac{\pi}{2}$ gives us a negative cos curve so other possible equation is
$y = -\frac{2\pi}{3}\cos(3x - \frac{\pi}{2})$

Another possible answer is
$y = \frac{2\pi}{3}\cos(3x + \frac{\pi}{2})$

b) Period $\frac{\pi}{2}$, sine graph shifted up 2 so a possible graph is
$y = \sin 4x + 2$
Now a sine curve shifted to the left $\frac{3\pi}{2}$ is a cos curve
Hence another possible curve is
$y = \cos(4x + \frac{3\pi}{2}) + 2$ or $y = \cos 4(x + \frac{3\pi}{8}) + 2$

P 2

P 3

2B

Q1
Q2
Q3
Q4

a) related angle $= \frac{\pi}{6}$ $\quad 0 \le 2x \le 4\pi$

$2x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$

$x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

b) related angle $\frac{\pi}{3}$ $\quad 0 \le \frac{x}{2} \le \pi$

$\frac{x}{2} = \frac{\pi}{3}, \frac{2\pi}{3}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

c) related angle $\frac{\pi}{4}$ $\quad 0 \le \frac{2x}{3} \le \frac{4\pi}{3}$

$\frac{2x}{3} = \frac{5\pi}{4}$

$x = \frac{15\pi}{8}$

d) related angle $\frac{\pi}{4}$ $\quad 0 \le 3x \le 6\pi$

$3x = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4}, \frac{17\pi}{4}, \frac{23\pi}{4}$

$x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{17\pi}{12}, \frac{23\pi}{12}$

e) related angle $\frac{\pi}{6}$ $\quad 0 \le \frac{x}{3} \le \frac{2\pi}{3}$

$\frac{x}{3} = \frac{5\pi}{6}$ not in domain

$\therefore$ no solution

f) related angle $\frac{\pi}{3}$ $\quad 0 \le \frac{3x}{2} \le 3\pi$

$\frac{3x}{2} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$

$x = \frac{2\pi}{9}, \frac{10\pi}{9}, \frac{14\pi}{9}$

g) related angle $\frac{\pi}{4}$ $\quad 0 \le 3x \le 6\pi$

$3x = \frac{3\pi}{4}, \frac{7\pi}{4}, \frac{11\pi}{4}, \frac{15\pi}{4}, \frac{19\pi}{4}, \frac{23\pi}{4}$

$x = \frac{\pi}{4}, \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{5\pi}{4}, \frac{19\pi}{12}, \frac{23\pi}{12}$

h) related angle $\frac{\pi}{6}$ $\quad 0 \le \frac{x}{2} \le \pi$

$\frac{x}{2} = \frac{\pi}{6}$

$x = \frac{\pi}{3}$

i) related angle $\frac{\pi}{3}$ $\quad 0 \le \frac{3x}{2} \le 3\pi$

$\frac{3x}{2} = \frac{2\pi}{3}, \frac{5\pi}{3}, \frac{8\pi}{3}$

$x = \frac{4\pi}{9}, \frac{10\pi}{9}, \frac{16\pi}{9}$

Q5

a) related angle $\frac{\pi}{6}$ $\qquad -\frac{2\pi}{3} \le x+\frac{\pi}{3} \le \frac{4\pi}{3}$

$x+\frac{\pi}{3} = -\frac{\pi}{6}, \frac{7\pi}{6}$

$x = -\frac{\pi}{2}, \frac{5\pi}{6}$

b) related angle $\frac{\pi}{4}$ $\qquad -\frac{5\pi}{3} \le x-\frac{2\pi}{3} \le \frac{\pi}{3}$

$x-\frac{2\pi}{3} = -\frac{3\pi}{4}, -\frac{5\pi}{4}$

$x = -\frac{\pi}{12}, -\frac{7\pi}{12}$

c) related angle $\frac{\pi}{6}$ $\qquad -\frac{\pi}{6} \le x+\frac{5\pi}{6} \le \frac{11\pi}{6}$

$x+\frac{5\pi}{6} = \frac{\pi}{6}, \frac{7\pi}{6}$

$x = -\frac{2\pi}{3}, \frac{\pi}{3}$

Q6

a) $\qquad -\frac{5\pi}{3} \le 2x+\frac{\pi}{3} \le \frac{7\pi}{3}$

$2x+\frac{\pi}{3} = \frac{\pi}{2}, -\frac{3\pi}{2}$

$2x = \frac{\pi}{6}, -\frac{11\pi}{6}$

$x = \frac{\pi}{12}, -\frac{11\pi}{12}$

b) $\frac{x}{3}-\frac{\pi}{2} = -\pi$ $\qquad -\frac{\pi}{3} \le \frac{x}{3} \le \frac{\pi}{3}$

not in domain $\qquad -\frac{5\pi}{6} \le \frac{x}{3}-\frac{\pi}{2} \le -\frac{\pi}{6}$

no solution

c) $2x+\frac{\pi}{3} = -\pi, 0, \pi, 2\pi$ $\qquad -2\pi \le 2x \le 2\pi$

$2x = -\frac{4\pi}{3}, -\frac{\pi}{3}, +\frac{2\pi}{3}, \frac{5\pi}{3},$ $\qquad -\frac{5\pi}{3} \le 2x+\frac{\pi}{3} \le \frac{7\pi}{3}$

$x = -\frac{2\pi}{3}, -\frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{6},$

d) $\qquad -3\pi \le 3x \le 3\pi$

related angle $\frac{\pi}{6}$ $\qquad -\frac{9\pi}{2} \le 3x-\frac{3\pi}{2} \le \frac{3\pi}{2}$

$3x-\frac{3\pi}{2} = -\frac{23\pi}{6}, -\frac{19\pi}{6}, -\frac{11\pi}{6}, -\frac{7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

$3x = -\frac{7\pi}{3}, -\frac{5\pi}{3}, -\frac{\pi}{3}, +\frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$

$x = -\frac{7\pi}{9}, -\frac{5\pi}{9}, -\frac{\pi}{9}, \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$

e) related angle $\frac{\pi}{3}$, $\qquad -\frac{4\pi}{3} \le 2x+\frac{2\pi}{3} \le \frac{8\pi}{3}$

$2x+\frac{2\pi}{3} = -\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$

$2x = -2\pi, -\frac{4\pi}{3}, 0, \frac{2\pi}{3}, 2\pi$

$x = -\pi, -\frac{2\pi}{3}, 0, \frac{\pi}{3}, \pi$

f) related angle $\frac{\pi}{3}$

$-\frac{\pi}{2} \le \frac{x}{2} \le \frac{\pi}{2}$

$-\frac{2\pi}{3} \le \frac{x}{2} - \frac{\pi}{6} \le \frac{\pi}{3}$

$\frac{x}{2} - \frac{\pi}{6} = -\frac{2\pi}{3}, \frac{\pi}{3}$

$\frac{x}{2} = -\frac{\pi}{2}, \frac{\pi}{2}$

$x = -\pi, \pi$

Q7 a)

$-4\pi \le 2x \le 4\pi$

$2x = \pm 4\pi, \pm 3\pi, \pm 2\pi, \pm \pi, 0$

$x = \pm 2\pi, \pm \frac{3\pi}{2}, \pm \pi, \pm \frac{\pi}{2}, 0$

b)

$-6\pi \le 3x \le 6\pi$

$3x = \pm \frac{11\pi}{2}, \pm \frac{9\pi}{2}, \pm \frac{7\pi}{2}, \pm \frac{5\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{\pi}{2}$

$x = \pm \frac{11\pi}{6}, \pm \frac{3\pi}{2}, \pm \frac{7\pi}{6}, \pm \frac{5\pi}{6}, \pm \frac{\pi}{2}, \pm \frac{\pi}{6}$

c)

$-4\pi \le 2x \le 4\pi$

$2x = \pm 4\pi, \pm 3\pi, \pm 2\pi, \pm \pi, 0$

$x = \pm 2\pi, \pm \frac{3\pi}{2}, \pm \pi, \pm \frac{\pi}{2}, 0$

d)

$-\pi \le \frac{x}{2} \le \pi$

$\frac{x}{2} = -\frac{\pi}{2}$

$x = -\pi$

e)

$-\frac{2\pi}{3} \le \frac{x}{3} \le \frac{2\pi}{3}$

$\frac{x}{3} = 0$

$x = 0$

f)

$-\frac{4\pi}{3} \le \frac{2x}{3} \le \frac{4\pi}{3}$

$\frac{2x}{3} = \pm \pi, 0$

$x = \pm \frac{3\pi}{2}, 0$

g)

$-4\pi \le 2x \le 4\pi$

$-\frac{13\pi}{3} \le 2x - \frac{\pi}{3} \le \frac{11\pi}{3}$

$2x - \frac{\pi}{3} = -\frac{7\pi}{2}, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}$

$2x = -\frac{19\pi}{6}, -\frac{7\pi}{6}, \frac{5\pi}{6}, \frac{17\pi}{6}$

$x = -\frac{19\pi}{12}, -\frac{7\pi}{12}, \frac{5\pi}{12}, \frac{17\pi}{12}$

h)

$-\frac{14\pi}{3} \le 2x - \frac{2\pi}{3} \le \frac{10\pi}{3}$

$2x - \frac{2\pi}{3} = -3\pi, -\pi, \pi, 3\pi$

$2x = -\frac{7\pi}{3}, -\frac{\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{3}$

$x = -\frac{7\pi}{6}, -\frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

i) $-\pi \le \frac{x}{2} \le \pi$

$-\frac{\pi}{4} \le \frac{x}{2} + \frac{3\pi}{4} \le \frac{7\pi}{4}$

$\frac{x}{2} + \frac{3\pi}{4} = 0, \pi$

$\frac{x}{2} = -\frac{3\pi}{4}, \frac{\pi}{4}$

$x = -\frac{3\pi}{2}, \frac{\pi}{2}$

Q8 It is a good execise to solve these equations algebraically and to check your answers graphically.

P1 a) $\cos(x + \frac{\pi}{4}) = \frac{1}{\sqrt{2}}$ $\quad \frac{\pi}{4} \le x + \frac{\pi}{4} \le \frac{9\pi}{4}$

related angle $\frac{\pi}{4}$

$x + \frac{\pi}{4} = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}$

$x = 0, \frac{3\pi}{2}, 2\pi$

b) $\sin(x - \frac{3\pi}{2}) = \frac{1}{2}$ $\quad -\frac{3\pi}{2} \le x - \frac{3\pi}{2} \le \frac{\pi}{2}$

related angle $\frac{\pi}{6}$

$x - \frac{3\pi}{2} = -\frac{7\pi}{6}, \frac{\pi}{6}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

c) $\tan(x + \frac{2\pi}{3}) = \frac{1}{\sqrt{3}}$ $\quad \frac{2\pi}{3} \le x + \frac{2\pi}{3} \le \frac{8\pi}{3}$

Related angle $\frac{\pi}{6}$

$x + \frac{2\pi}{3} = \frac{7\pi}{6}, \frac{13\pi}{6}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$

Q2 a) related angle $\frac{\pi}{4}$ $\quad 0 \le 2x \le 4\pi$

$2x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4}, \frac{15\pi}{4}$

$x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

b) $\cos\frac{x}{2}(4\cos^2\frac{x}{2} - 3) = 0$ $\quad 0 \le x \le \pi$

$\cos\frac{x}{2} = 0$ $\quad \cos^2\frac{x}{2} = \frac{3}{4}$

$\cos\frac{x}{2} = \pm\frac{\sqrt{3}}{2}$

$\frac{x}{2} = \frac{\pi}{2}$ $\quad \frac{x}{2} = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

e) $0 \le 2x \le 4\pi$

$$\sin 2x = \frac{2}{\sin 2x} - 1$$

$$\sin^2 2x + \sin 2x - 2 = 0$$

$$(\sin 2x + 2)(\sin 2x - 1) = 0$$

$\sin 2x = -2$ no solution

$\sin 2x = 1$

$2x = \frac{\pi}{2}, \frac{5\pi}{2}$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

P3 a) Period is $\frac{2\pi}{3}$ so $a = 3$ and shifted to the right $\frac{\pi}{3}$

$$y = \sin 3\left(x - \frac{\pi}{3}\right)$$
$$= \sin(3x - \pi)$$

b) Period is π so $a = 2$ shifted to the left $\frac{\pi}{6}$

$$y = \sin 2\left(x + \frac{\pi}{6}\right)$$
$$= \sin\left(2x + \frac{\pi}{3}\right)$$

2C

Q1 a) initial means when $t=0$

$x = 3\cos 0$

$= 3$

b) $3 = 3\cos \pi t$

$\pi t = 2\pi$

$t = 2$ secs

c) $-1 \le \cos \pi t \le 1$

so $-3 \le 3\cos \pi t \le 3$

d) $x = 0$

e) Period $= \frac{2\pi}{\pi} = 2$

Q2 a) amplitude $= 80$

$\therefore k = 80$

period $24 = \frac{2\pi}{a}$

$\therefore a = \frac{\pi}{12}$

when $t=0$ $x=0$

$\therefore$ equation $x = 80\sin\frac{\pi t}{12}$

b) $40 = 80\sin\frac{\pi t}{12}$

$\sin\frac{\pi t}{12} = \frac{1}{2}$

$\frac{\pi t}{12} = \frac{\pi}{6}$ $\qquad$ $\frac{\pi t}{12} = \frac{7\pi}{6}$

$t = 2$ or $t = 14$

c) $x = 80\sin\frac{10\pi}{12}$

$= 80 \times \frac{1}{2}$

$= 40$ m

Q3 a) amplitude $= 6$

period $= \frac{2\pi}{2} = \pi$

b) This motion is initially at $x = 6\cos(-\frac{\pi}{3}) = 3$ while $x = 6\cos 2t$ is initially at $x = 6$. Both oscillate between -6 and 6 just the times reaching various positions have shifted.

c)

d) $x = 6$ $\quad$ $x = 3$

e) amplitude and period same

f) 4 times each.

This can be seen from the graph or by solving $x=0$ for both equations

g) you would the line $x = -3$ on an accurate graph and read off the values.

for $x = 6\cos(2t - \frac{\pi}{3}) = -3$

$\cos(2t - \frac{\pi}{3}) = -\frac{1}{2}$

$2t - \frac{\pi}{3} = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}$

$t = \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}, \ldots$

for $x = 6\cos 2t = -3$

$\cos 2t = -\frac{1}{2}$

$t = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \ldots$

The above is checking the values

h) Period is the same but this particle oscillates between 0 and 12

Q4 a)

b) This the period $= \frac{2\pi}{\pi} = 2$ secs.

c) Period would change $h = 28 + 14\sin at$, where $a > \pi$

Q5 a)

b) $T = 10 + 20 = 30°C$ highest temp in July.

c) $T = -10 + 20 = 10°C$ lowest temp in January.

P1 Since particle at origin at $t = \frac{\pi}{6}$ and $\frac{5\pi}{12}$ half period $= \frac{\pi}{4}$

period is $\frac{\pi}{2}$ hence $a = 4$

as period $= \frac{2\pi}{a} = \frac{\pi}{2}$

$k = 2$ as particle oscillates between -2 and 2

Now $x = 2\cos(4t + b) = 0$ at $t = \frac{\pi}{6}$

$\cos(\frac{2\pi}{3} + b) = 0$

$\frac{2\pi}{3} + b = \frac{\pi}{2}$

$b = -\frac{\pi}{6}$

Hence $x = 2\cos(4t - \frac{\pi}{6})$

P2

Review

Q1 a) $\frac{2\pi}{2} = \pi$, $[-2, 2]$

b) $\frac{2\pi}{\frac{1}{3}} = 6\pi$, $[-2, 2]$

c) $\frac{\pi}{\frac{1}{2}} = 2\pi$, all real y or $(-\infty, \infty)$

d) 2π, $[-2, 2]$

e) $\frac{2\pi}{3}$, $[2, 2]$

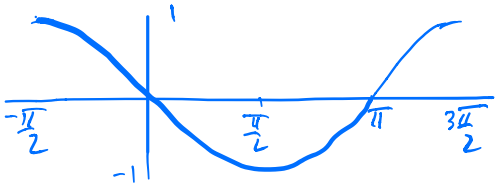
f) $\frac{2\pi}{2} = \pi$, $[3, 5]$

g) $\frac{2\pi}{\frac{1}{2}} = 4\pi$, $[4, 6]$

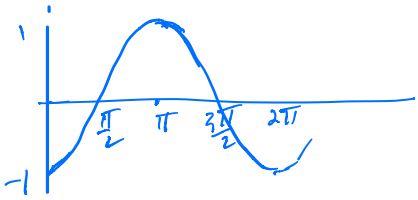
h) $\frac{2\pi}{2} = \pi$ $[-3, 3]$

i) $\frac{2\pi}{\frac{1}{3}} = 6\pi$, $[1, 3]$

Q2 a) $y = \cos(x + \frac{\pi}{2})$
$= -\sin x$



b) $y = \cos(x - \pi)$
$= -\cos x$



c) $y = \cos(x - 2\pi)$
$= \cos x$

Q3 Period is π so $a = 2$
Range $[-3, 3]$ so $k = 3$
$y = 3\sin 2x$

Q4 Period is 4π so $a = \frac{1}{2}$
Range $[-2, 2]$
$y = -2\cos\frac{x}{2}$

Q5 Period is 2π
Range $[-6, 0]$
sine curve shifted down 3
$y = 3\sin x - 3$

Q6 Period π
Range $[-2, 4]$ $k = -3$
$y = -3\cos 2x$ shifted up 1 so
$y = -3\cos 2x + 1$

Q7 Period $\frac{\pi}{2}$, negative tan graph
$y = -\tan 2x$

Q8

Q9

Q10

Q11

Q12

Q13a) No solution $0 \le x \le 2\pi$

$0 \le \frac{x}{3} \le \frac{2\pi}{3}$

b) $\sin 2x = -\frac{1}{2}$ $\quad 0 \le 2x \le 4\pi$

related angle $= \frac{\pi}{6}$

$2x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}, \frac{23\pi}{6}$

$x = \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12}$

c) related angle $\frac{\pi}{3}$ $\quad 0 \le x \le 2\pi, \ \frac{\pi}{6} \le x + \frac{\pi}{6} \le 2\pi + \frac{\pi}{6}$

$x + \frac{\pi}{6} = \frac{\pi}{3}, \quad x + \frac{\pi}{6} = 2\pi - \frac{\pi}{3}$

$x = \frac{\pi}{6}, \quad x = \frac{3\pi}{2}$

d) related angle is $\frac{\pi}{6}$ $\quad -\frac{\pi}{6} \le 2x - \frac{\pi}{6} \le 4\pi - \frac{\pi}{6}$

$\therefore \ 2x - \frac{\pi}{6} = -\frac{\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}$

$2x = 0, \frac{\pi}{3}, 2\pi, \frac{7\pi}{3}, 4\pi$

$x = 0, \frac{\pi}{6}, \pi, \frac{7\pi}{6}, 2\pi$

Q14 $\quad$ domain $-\pi \le x \le \pi$

a) related angle $\frac{\pi}{3}$ $\quad -2\pi \le 2x \le 2\pi$

$2x = -\frac{\pi}{3}, -\frac{5\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$

$x = -\frac{\pi}{6}, -\frac{5\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

b) related angle $\frac{\pi}{4}$, $\quad -2\pi \le 2x \le 2\pi$

$2x = -\frac{\pi}{4}, -\frac{5\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$

$x = -\frac{\pi}{8}, -\frac{5\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}$

c) related angle $\frac{\pi}{4}$, $\quad -\frac{\pi}{3} \le \frac{x}{3} \le \frac{\pi}{3}$

$\frac{x}{3} = \frac{\pi}{4}$

$x = \frac{3\pi}{4}$

d) related angle $\frac{\pi}{4}$, $\quad -\frac{3\pi}{4} \le x + \frac{\pi}{4} \le \frac{5\pi}{4}$

$x + \frac{\pi}{4} = -\frac{\pi}{4}, \frac{3\pi}{4}$

$x = -\frac{\pi}{2}, \frac{\pi}{2}$

e) related angle $\frac{\pi}{3}$ $\quad -2\pi \le 2x \le 2\pi$

$2x = -\frac{\pi}{3}, -\frac{4\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{3}$

$x = -\frac{\pi}{6}, -\frac{2\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{6}$

Q15 a) max displacement 8m.

b) period $= \frac{2\pi}{\pi} = 2$

c)

Q16 a) $h = 1.6 - 0.4 = 1.2$ m

$h = 1.6 + 0.4 = 2$ m

b) $2 = 1.6 + 0.4 \cos 2\pi t$

$\cos 2\pi t = 1$

$t = 0, 1, 2,$

so every 1 sec at highest point

Q17 a) 4

b) period $= \frac{2\pi}{0.0172} = 365.30$

approx 365 days

formula gives full cycle for the year

c) $H = 4 + 12 = 16$ hours and it occurs when

$\cos 0.0172t = -1$

$0.0172t = \pi$

183rd day

2nd July

Q18 a) high tide $= 8 + 12 = 20$m

low tide $= -8 + 12 = 4$m

b)

c) note period $= \frac{2\pi}{\frac{\pi}{6}} = 12$ hours

So every 12 hours tide is in the same position repeating motion.