# 1. Algebraic Techniques

## Exercise 1A

### Expanding and factorising

**Q1**

(a) $4x^2 + 4x + 1$

(b) $9x^2 - 24x + 16$

(c) $9x^2y^2 - 24xy + 16$

(d) $9x^2 - 24xy + 16y^2$

(e) $x^2 + 2 + \dfrac{1}{x^2}$

(f) $x^2 - \dfrac{1}{x^2}$

(g) $6x^2 - 5x - 4$

(h) $4x^2 - 9y^2$

(i) $6x^2 - 5xy - 4y^2$

**Q2**

(a) $(x+3)^2$

(b) $(x+5)(x-3)$

(c) $(2x+1)(2x-5)$

(d) $3(4x+3y)(4x-3y)$

(e) $(5+x+y)(5-x-y)$

(f) $(2x+3)(x-5)$

(g) $(x+y+2)(x-y)$

(h) $(x-3)(1+x)$

(i) $(x^2+y^2)(x+y)(x-y)$

**Q3**

(a) $(x-1)(1+x+x^2) = x + x^2 + x^3 - 1 - x - x^2$
$= x^3 - 1$

(b) From part (a), we have

$$(x-1)(1+x+x^2) = x^3 - 1.$$

Hence $1 + x + x^2 = \dfrac{x^3-1}{x-1}$. We have $x \neq 1$ because the denominator cannot be zero.

**P1**

(a) $(4 - 9x^2)(4 + 9x^2) = (2 - 3x)(2 + 3x)(4 + 9x^2)$

(b) $(a+2)(a^2-4) = (a+2)(a+2)(a-2)$
$= (a+2)^2(a-2)$

(c) $(2x-3)(3x+2)$

(d) $(2x-3y)(3x+2y)$

**P2**

(a) $(x-1)(x^2+x+1)$

(b) $(3x-2)(9x^2+6x+4)$

(c) $(2x+1-2)\left((2x+1)^2 + 2(2x+1) + 4\right)$
$= (2x-1)(4x^2+8x+7)$

**P3**

(a) $x^3 - 3x^2y + 3xy^2 - y^3$

(b) $x^3 + 3x^2y + 3xy^2 + y^3$

**P4**

(a) RHS
$= (x-1)(1 + x + x^2 + x^3 + \cdots + x^{n-2} + x^{n-1})$
$= x + x^2 + x^3 + x^4 + \cdots + x^{n-1} + x^n$
$\quad - 1 - x - x^2 - x^3 - \cdots - x^{n-2} - x^{n-1}$
$= -1 + x^n = x^n - 1 = \text{LHS}$

(b) Divide both sides of a) by $x - 1$

$$\therefore \quad \frac{x^n - 1}{x - 1} = 1 + x + x^2 + \cdots + x^{n-1}$$

(c) $x = 2$, $n = 100$ using (b)

$$\frac{2^{100} - 1}{2 - 1} = 2^{100} - 1$$

**P5**

$z^6 + 1 = (z^2)^3 + 1$
$= (z^2+1)(z^4 - z^2 + 1)$
$= (z^2+1)(z^4 + 2z^2 + 1 - 3z^2)$
$= (z^2+1)((z^2+1)^2 - 3z^2)$
$= (z^2+1)(z^2 - z\sqrt{3} + 1)(z^2 + z\sqrt{3} + 1)$

**P6**

RHS
$= a^3 + ab^2 + ac^2 - a^2b - abc - a^2c + a^2b + b^3 + bc^2$
$\quad - ab^2 - b^2c - abc + a^2c + b^2c + c^3 - abc - bc^2 - ac^2$
$= a^3 + b^3 + c^3 - 3abc$

# Exercise 1B
## Algebraic fractions

### Q1

(a) $\frac{6x}{5}$

(b) $\frac{9x+22x}{30} = \frac{31x}{30}$

(c) $\frac{3x-1-7+2x}{3} = \frac{5x-8}{3}$

(d) $\frac{4(5x+3)-3(1-4x)}{24} = \frac{32x+9}{24}$

(e) $\frac{17}{8x}$

(f) $\frac{5-8}{4x} = -\frac{3}{4x}$

(g) $\frac{x(x+1)-x(x-1)}{(x-1)(x+1)}$

(h) $\frac{3x+5-(x+7)}{x-2} = \frac{2x-2}{x-2} = \frac{2(x-1)}{x-2}$

(i) $\frac{2x+x-2}{x(x-2)} = \frac{3x-2}{x(x-2)}$

(j) $\frac{9+10x}{12x^2}$

(k) $\frac{5(x+2)+2(x-2)}{x(x-2)(x+2)} = \frac{7x+6}{x(x-2)(x+2)}$

(l) $\frac{3}{x(x-4)} - \frac{2}{(x-2)(x-1)}$
$= \frac{3(x-2)(x-1)-2(x)(x-4)}{x(x-4)(x-2)(x-1)}$
$= \frac{3x^2-9x+6-2x^2+8x}{x(x-4)(x-2)(x-1)}$
$= \frac{x^2-x+6}{x(x-4)(x-2)(x-1)}$

### Q2

(a) $\frac{5x}{5y} \times \frac{3y}{20x^2} = \frac{3}{20x}$

(b) $\frac{5x}{6} \times \frac{9}{15xy} = \frac{1}{2y}$

(c) $\frac{x-2}{x+3} \times \frac{(x-3)(x+2)}{(x-3)^2} = \frac{(x+2)(x-2)}{(x+3)(x-3)}$

(d) $\frac{(x-3)(x+3)}{(x-5)(x+3)} \times \frac{3(x-2)(x+2)}{(x-2)(x-3)} = \frac{3(x+2)}{x-5}$

### Q3

(a) $\frac{2x(2x+3y)}{8x^2} = \frac{2x+3y}{4x}$

(b) $\frac{y-x}{xy} \times \frac{1}{x-y} = \frac{(y-x)}{xy} \times \frac{-1}{(y-x)}$
$= -\frac{1}{xy}$

(c) $\left(\frac{y+x}{xy}\right) \times \frac{(y-x)}{xy} = \frac{y^2-x^2}{x^2y^2}$

### Q4

(a) $\frac{x^2-1}{x^2+1}$

(b) $\frac{x+1+x}{x+1-x} = \frac{2x+1}{1}$

### Q5

(a) $\frac{1}{x+1}$ $\left(\text{multiplying by } \frac{x}{x}\right)$

(b) $\frac{y+x}{y-x}$ $\left(\text{multiplying by } \frac{xy}{xy}\right)$

(c) $\frac{y^2+x^2}{y^2-x^2}$ $\left(\text{multiplying by } \frac{xy}{xy}\right)$

(d) $\frac{2(x+3)+x}{3(x+3)-x} = \frac{3(x+2)}{2x+9}$
$\left(\text{multiplying by } \frac{x(x+3)}{x(x+3)}\right)$

### Q6

(a) $\frac{1-x}{x-1} = -1$

(b) $\frac{(t-3)(t+3)}{3(t-3)} = \frac{t+3}{3}$

(c) $\frac{3(x-1)-2(x+2)}{(x+2)(x-2)(x-1)}$
$= \frac{x-7}{(x+2)(x-2)(x-1)}$

(d) $\frac{2-(x+3)-5(x-3)}{(x-3)(x+3)}$
$= \frac{-6x+14}{(x-3)(x+3)} = \frac{2(7-3x)}{(x-3)(x+3)}$

### Q7

(a) $\frac{1}{a} + \frac{1}{b} = \frac{b+a}{ab} = \frac{5}{3}$

(b) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{b^2+a^2}{a^2b^2} = \frac{(a+b)^2-2ab}{a^2b^2}$
$= \frac{25-6}{9} = \frac{19}{9}$

(c) $\frac{b+a}{ab(a+b)} = \frac{1}{ab} = \frac{1}{3}$

### Q8

$\frac{x(x+2)+1}{(x+1)(x+2)} = \frac{x^2+2x+1}{(x+1)(x+2)}$
$= \frac{(x+1)^2}{(x+1)(x+2)} = \frac{x+1}{x+2}$

**P1**

$$\frac{1+n^2-1-n(n+1)}{n} = \frac{n^2-n^2-n}{n}$$
$$= -1$$

or, simplifying first two terms

$$\frac{1+n^2-1}{n} - (n+1) = \frac{n^2}{n} - n - 1$$
$$= n - n - 1 = -1$$

**P2**

(a) $$\frac{a+\frac{1}{a}}{a-\frac{1}{a}} \times \frac{a}{a} = \frac{a^2+1}{a^2-1}$$

(b) $$\frac{\frac{a}{b}+\frac{b}{a}}{\frac{a}{b}-\frac{b}{a}} \times \frac{ab}{ab} = \frac{a^2+b^2}{a^2-b^2}$$

**P3**

$$q = \frac{1}{1-p} = \frac{1}{1-\frac{1}{a}} \times \frac{a}{a} = \frac{a}{a-1} \quad (1)$$

$$r = \frac{q}{q-1} = \frac{\frac{a}{a-1}}{\frac{a}{a-1}-1} \times \frac{a-1}{a-1}$$

now substitute (1)

$$= \frac{a}{a-(a-1)} = \frac{a}{1}$$

**P4**

(a) To find the average of 2 numbers we add them and divide by 2.

$$\frac{\frac{1}{a}+\frac{1}{b}}{2} \times \frac{ab}{ab} = \frac{b+a}{2ab} = \frac{a+b}{2ab}$$

or halfway between

$$\frac{1}{a} = \frac{b}{ab} \text{ and } \frac{1}{b} = \frac{a}{ab} \text{ is } \frac{a+b}{2ab}$$

(b) Average of $\frac{1}{2}$ and $\frac{1}{10}$

$$= \frac{2+10}{2\times 2\times 10} = \frac{12}{40} = \frac{3}{10}$$

note we could also find average

$$\frac{1}{2} = \frac{5}{10} \text{ so average } \frac{6}{10} \div 2 = \frac{3}{10}$$

(c) According to Bob the average of $a$ and $b$ is $\frac{a+b}{2}$ and find reciprocal $\frac{2}{a+b}$

(d) $$x - y = \frac{a+b}{2ab} - \frac{2}{a+b} \quad \text{from (a)}$$
$$= \frac{(a+b)^2-4ab}{2ab(a+b)}$$
$$= \frac{(a-b)^2}{2ab(a+b)}$$

Since $a$ and $b$ are positive $x - y > 0$

$$\therefore \quad x > y$$

$\therefore$ Bob's average is less than actual average $x$.

**P5**

(a)

$$\frac{1}{u} = \frac{1}{f} - \frac{1}{v} \quad (1)$$
$$\frac{1}{u} = \frac{v-f}{fv}$$
$$u = \frac{fv}{v-f} \quad (2)$$

(b) $v - f \neq 0$ is $v \neq f$ also in equation if $v = f$ RHS$= 0$

$$\implies \frac{1}{u} = 0 \text{ is not valid}$$

(c) $$x_1x_2 = (u-f)(v-f)$$
$$= uv - fv - uf + f^2$$

substituting for $fv$ from (a)

$$= uv - uf - u(v-f) + f^2$$
$$= f^2$$

# Exercise 1C

## Indices

### Q1

(a) $1$

(b) $7$

(c) $5$

(d) $\left(8^{\frac{1}{3}}\right)^2 = 2^2 = 4$

(e) $\left(9^{\frac{1}{2}}\right)^3 = 3^3 = 27$

(f) $\left(\left(\frac{125}{64}\right)^{\frac{1}{3}}\right)^2 = \left(\frac{5}{4}\right)^2 = \frac{25}{16}$

(g) $\left(-8^{\frac{1}{3}}\right)^2 = (-2)^2 = 4$

### Q2

(a) $\frac{1}{9^{\frac{1}{2}}} = \frac{1}{3}$

(b) $\frac{1}{5^2} = \frac{1}{25}$

(c) $3^3 = 27$

(d) $\left(\frac{5}{2}\right)^2 = \frac{25}{4}$

(e) $\left\{\left(\frac{1}{25}\right)^{-\frac{1}{2}}\right\}^3 = 5^3 = 125$

(f) $\frac{1}{4^{\frac{1}{2}}} = \frac{1}{2}$

(g) $\left(125^{-\frac{1}{3}}\right)^2 = \left(\frac{1}{5}\right)^2 = \frac{1}{25}$

(h) $(2^3)^{-3} \times 2^8 = 2^{-9} \times 2^8 = 2^{-1} = \frac{1}{2}$

### Q3

(a) $2 \times 1 + 1 = 3$

(b) $3^{2x} \times 3^2 = 3^{2x+2}$

(c) $3^{3x+1}$

(d) $36x^6y^8$

(e) $4x^{-6} = \frac{4}{x^6}$

(f) $4x^7$

### Q4

(a) $2^n \times 2^{2n} \times 2^{3n} \times 2^{4n} = 2^{10n}$

(b) $\frac{7^{-2n} \times 7}{7^{3n}} = 7^{1-5n}$

(c) $\frac{3^n \times 3^{2n+2}}{3^{3n-6}} = 3^8$

(d) $\frac{(3^n - 1)(3^n + 1)}{(3^n - 1)} = 3^n + 1$

(e) $\frac{5^n(5^n - 1)}{5^n - 1} = 5^n$

### Q5

(a) $2^x = 2^4$

$x = 4$

(b) $2^{2x+2} = 2^4$

$2x + 2 = 4$

$x = 1$

(c) $2^{2x+2} = 2^0$

$x = -1$

or $4^{x+1} = 4^0$

$x + 1 = 0$

$x = -1$

(d) $2^{2x+2} = 2^{-3}$

$2x + 2 = -3$

$2x = -5$

$x = -\frac{5}{2}$

(e) $3^{3x+3} = 3 \times 3^{2x-2}$

$3x + 3 = 2x - 1$

$x = -4$

(f) $\frac{1}{x^2} = 49$

$x^2 = \frac{1}{49}$

$x = \pm\frac{1}{7}$

### Q6

(a) $3^7(3 - 1) = 2 \times 3^7$

(b) $2^n(1 + 2) = 3 \times 2^n$

(c) $3^n(3^n - 1)$

(d) $3^n(2^n - 1)$

(e) $3^n(3^{2n} - 1) = 3^n(3^n + 1)(3^n - 1)$

### P1

(a) $2^{-4x} = 2^{-5}$

$-4x = -5$

$x = \frac{5}{4}$

(b) $2^{3x} = 2^{2x+2}$
$3x = 2x + 2$
$x = 2$

(c) $(2^4)^{2-n} = \dfrac{1}{2^{3x}}$
$2^{8-4n} = 2^{-3x}$
$8 - 4n = -3x$
$x = 8$

(d) $2^{3x} = 2\left(2^{x+3}\right)$
$2^{3x} = 2^{x+4}$
$3x = x + 4$
$2x = 4$
$x = 2$

(e) $2^{5x} = 2^{x+4}(2 - 1)$
$2^{5x} = 2^{x+4}$
$5x = x + 4$
$4x = 4$
$x = 1$

(f) $2^{2x} - 2 \times 2^x - 8 = 0$
Let $u = 2^x$
$u^2 - 2u - 8 = 0$
$(u - 4)(u + 2) = 0$
$u = 4,\ -2$

| $u = 4$ | $u = -2$ |
|---|---|
| $2^x = 4$ | $2^x = -2$ |
| $x = 2$ | No solution |

## P2

(a) $\dfrac{3^x \times 4^x}{8^x} \times \dfrac{6^x}{3^x} = \dfrac{6^x}{2^x} = 3^x$

(b) $\sqrt{2^{4x} \times 2^{4x} \times 2^{12x}}$
$= \sqrt{2^{20x}} = \left(2^{20x}\right)^{\frac{1}{2}}$
$= 2^{10x}$

Alternatively:

$\sqrt{2^{4x}} \times \sqrt{4^{2x}} \times \sqrt{8^{4x}}$
$= 2^{2x} \times 4^x \times 8^{2x}$
$= 2^{2x} \times 2^{2x} \times 2^{6x}$
$= 2^{10x}$

## P3

half of $2^{200} = \dfrac{2^{200}}{2}$
$= 2^{199}$
$\therefore\ x = 199$

## P4

$5^n(5 + 1) = 6 \times 5^n$
$\dfrac{5^{100}(5 + 1)}{6} = 5^{100}$

## P5

(a) $(5^x - 2^y)(5^x + 2^y)$ Difference of squares

(b) $2^{4x}\left(1 - 2^{2x}\right) = 2^{4x}(1 - 2^x)(1 + 2^x)$

## P6

6 lots of $6^6 = 6 \times 6^6 = 6^7$

## P7

| | |
|---|---|
| $2^a + 3^b = 17$ | (1) |
| $2 \times 2^a - 3 \times 3^b = -11$ | (2) |
| $2 \times 2^a + 2 \times 3^b = 34$ | $2 \times (1)$ |
| $5 \times 3^b = 45$ | subtract |
| $3^b = 9$ | |
| $3^b = 3^2$ | |
| $b = 2$ | |
| $2^a + 3^2 = 17$ | sub in (1) |
| $2^a = 8$ | |
| $a = 3$ | |

# Exercise 1D

## Surds

### Q1

(a) $\sqrt{9\times5}+\sqrt{16\times5}-\sqrt{9\times4\times2}$
$=3\sqrt{5}+4\sqrt{5}-6\sqrt{2}$
$=7\sqrt{5}-6\sqrt{2}$

(b) $3\sqrt{4\times6}+6\sqrt{25\times6}-5\sqrt{9\times6}$
$=3\times2\sqrt{6}+6\times5\sqrt{6}-5\times3\sqrt{6}$
$=6\sqrt{6}+30\sqrt{6}-15\sqrt{6}$
$=21\sqrt{6}$

(c) $\dfrac{4+2\sqrt{3}}{2}=\dfrac{2(2+\sqrt{3})}{2}=2+\sqrt{3}$

(d) $\dfrac{2\sqrt{6}\times3\sqrt{7}}{12\sqrt{21}}=\dfrac{\sqrt{2}}{2}$

### Q2

(a) $5-2=3$

(b) $\sqrt[3]{8\times3}=2\sqrt[3]{3}$

(c) $\sqrt[3]{1000\times2}=10\sqrt[3]{2}$

(d) $\sqrt[5]{32\times2}=2\sqrt[5]{2}$

### Q3

(a) $a\sqrt{b}$

(b) $a^2b$

(c) $a^2b$

(d) $a^2b\sqrt{ab}$

### Q4

(a) $\dfrac{1}{2\sqrt{5}}\times\dfrac{\sqrt{5}}{\sqrt{5}}=\dfrac{\sqrt{5}}{10}$

(b) $\dfrac{5\sqrt{2}}{3\sqrt{2}}\times\dfrac{\sqrt{10}}{\sqrt{10}}=\dfrac{5\times\sqrt{2}\times\sqrt{2}\sqrt{5}}{3\times10}=\dfrac{\sqrt{5}}{3}$

(c) $\dfrac{5\sqrt{3}}{3\sqrt{5}}\times\dfrac{\sqrt{5}}{\sqrt{5}}=\dfrac{\sqrt{15}}{3}$

(d) $\dfrac{\sqrt{2}}{5-\sqrt{2}}\times\dfrac{5+\sqrt{2}}{5+\sqrt{2}}=\dfrac{\sqrt{2}(5+\sqrt{2})}{25-2}=\dfrac{5\sqrt{2}+2}{23}$

(e) $\dfrac{5+\sqrt{2}}{5-\sqrt{2}}\times\dfrac{5+\sqrt{2}}{5+\sqrt{2}}=\dfrac{25+10\sqrt{2}+2}{25-2}$
$=\dfrac{27+10\sqrt{2}}{23}$

(f) $\dfrac{1}{\sqrt{3}+1}\times\dfrac{\sqrt{3}-1}{\sqrt{3}-1}+\dfrac{\sqrt{3}-1}{4}$
$=\dfrac{\sqrt{3}-1}{3-1}+\dfrac{\sqrt{3}-1}{4}$
$=\dfrac{\sqrt{3}-1}{2}+\dfrac{\sqrt{3}-1}{4}$
$=\dfrac{2\sqrt{3}-2+\sqrt{3}-1}{4}$
$=\dfrac{3\sqrt{3}-3}{4}$

(g) $\dfrac{42}{3\sqrt{7}}-\dfrac{6}{\sqrt{7}+2}\times\dfrac{\sqrt{7}-2}{\sqrt{7}-2}$
$=\dfrac{14}{\sqrt{7}}\times\dfrac{\sqrt{7}}{\sqrt{7}}-\dfrac{6(\sqrt{7}-2)}{7-4}$
$=2\sqrt{7}-2(\sqrt{7}-2)$
$=4$

### Q5

(a) $\dfrac{a\sqrt{b}}{c\sqrt{d}}\times\dfrac{\sqrt{d}}{\sqrt{d}}=\dfrac{a\sqrt{bd}}{cd}$

(b) $\dfrac{1}{a+\sqrt{b}}\times\dfrac{a-\sqrt{b}}{a-\sqrt{b}}=\dfrac{a-\sqrt{b}}{a^2-b}$

(c) $\dfrac{1}{\sqrt{a}}\times\dfrac{\sqrt{a}}{\sqrt{a}}-\dfrac{1}{\sqrt{b}}\times\dfrac{\sqrt{b}}{\sqrt{b}}=\dfrac{\sqrt{a}}{a}-\dfrac{\sqrt{b}}{b}$

(d) $\dfrac{1}{a\sqrt{b}-c}\times\dfrac{a\sqrt{b}+c}{a\sqrt{b}+c}=\dfrac{a\sqrt{b}+c}{a^2b-c^2}$

(e) $\dfrac{1}{a\sqrt{b}+c\sqrt{d}}\times\dfrac{a\sqrt{b}-c\sqrt{d}}{a\sqrt{b}-c\sqrt{d}}=\dfrac{a\sqrt{b}-c\sqrt{d}}{a^2b-c^2d}$

### Q6

(a) $4-5=-1$

(b) $25+10\sqrt{3}+3=28+10\sqrt{3}$

(c) $25-20\sqrt{3}+12=37-20\sqrt{3}$

(d) $2+2\sqrt{6}+3=5+2\sqrt{6}$

(e) $1+\dfrac{2}{\sqrt{2}}+\dfrac{1}{2}=\dfrac{3}{2}+\sqrt{2}$

(f) $3-2+\dfrac{1}{3}=\dfrac{4}{3}$

### Q7

(a) $3+2\sqrt{4\times5}$ $\quad a=3,\ b=20$

(b) $4-12\sqrt{5}+45$ $\quad a=49,\ b=-12$

(c) $12-12\sqrt{18}+9\times6=12-36\sqrt{2}+54$ $\quad a=66,\ b=-36$

(d) $1-6\sqrt{2}+18-(4+12\sqrt{2}+18)=-3-18\sqrt{2}$
$a=-3,\ b=-18$

### Q8

(a) $\sqrt{x} = 3\sqrt{2} + 2\sqrt{2}$

$= 5\sqrt{2}$

$= \sqrt{25 \times 2}$

$= \sqrt{50}$

$\therefore \ x = 50$

(b) $x\sqrt{2} = 3\sqrt{2} + 6\sqrt{2} - 4\sqrt{2}$

$= 5\sqrt{2}$

$\therefore \ x = 5$

### Q9

(a) $x + \dfrac{1}{x} = \sqrt{3} + \sqrt{2} + \dfrac{1}{\sqrt{3}+\sqrt{2}} \times \dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$= \sqrt{3} + \sqrt{2} + \sqrt{3} - \sqrt{2}$

$= 2\sqrt{3}$

Now $\left(x + \dfrac{1}{x}\right)^2 = x^2 + 2 + \dfrac{1}{x^2}$

$\therefore \ (2\sqrt{3})^2 = x^2 + 2 + \dfrac{1}{x^2}$

$\therefore x^2 + \dfrac{1}{x^2} = 12 - 2$

$= 10$

(b) $x + \dfrac{1}{x} = \sqrt{5} - 2 + \dfrac{1}{\sqrt{5}-2} \times \dfrac{\sqrt{5}+2}{\sqrt{5}+2}$

$= \sqrt{5} - 2 + \sqrt{5} + 2 = 2\sqrt{5}$

$x - \dfrac{1}{x} = \sqrt{5} - 2 - (\sqrt{5} + 2) = -4$

$\therefore \ x^2 - \dfrac{1}{x^2} = \left(x + \dfrac{1}{x}\right)\left(x - \dfrac{1}{x}\right)$

$= (2\sqrt{5})(-4) = -8\sqrt{5}$

### Q10

LHS $= x^2 + 6x + 1$

$= (2\sqrt{2} - 3)^2 + 6(2\sqrt{2} - 3) + 1$

$= 8 - 12\sqrt{2} + 9 + 12\sqrt{2} - 18 + 1$

$= 0$

$\therefore \ 2\sqrt{2} - 3$ satisfies $x^2 + 6x + 1 = 0$

### P1

$\dfrac{4}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}} - \dfrac{1}{9-4\sqrt{5}} \times \dfrac{9+4\sqrt{5}}{9+4\sqrt{5}}$

$= \dfrac{4(2-\sqrt{5})}{-1} - \dfrac{9+4\sqrt{5}}{81-80}$

$= 4\sqrt{5} - 8 - 9 - 4\sqrt{5} = -17$

### P2

(a) $\dfrac{1}{\sqrt{n}+\sqrt{n+1}} \times \dfrac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n+1}-\sqrt{n}}$

$= \dfrac{\sqrt{n+1}-\sqrt{n}}{1}$

(b) Using (a) in each term

$\sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} +$

$\cdots + \sqrt{99} - \sqrt{98} + \sqrt{100} - \sqrt{99}$

$= -1 + \sqrt{100} = 10 - 1 = 9$

### P3

$xy\left(\dfrac{x}{y} - 2 + \dfrac{y}{x}\right) = x^2 - 2xy + y^2$ rational number

## Chapter Review

### R1

In each equation let the unknown be $x$

(a) Let Joel's age be $x$, then Zach's age is $x+4$

$x + x + 4 = 16$

$2x = 12$

$x = 6$

$\therefore$ Joel's age is 6 and Zach's age is 10

(b) $x - \frac{x}{4} = 12$

$\frac{3}{4}x = 12$

$3x = 48$

$x = 16$

(c) Let cost of a card be $x$ so cost of book is $\$6x$

$x + 6x = 28$

$7x = 28$

$x = 4$

$\therefore$ Card cost is \$4 and book \$24

(d) Let Jeremy's age be $x$ years old, so Natalie is $5x$ years old. Hence, their ages in 8 years are $x+8$ and $5x+8$ respectively.

$3(x+8) = 5x + 8$

$2x = 16$

$x = 8$

So Natalie is 40 years old now.

(e) If there are $x$ correct answers, Glen left out 6 questions so incorrect is $24 - x$

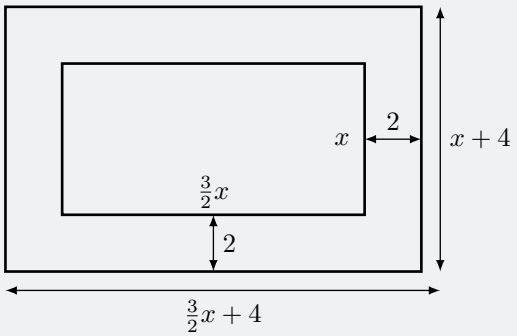
$2x - 3(24 - x) = 28$

$5x = 100$

$x = 20$

20 correct answers

(f) Let the width be $x$. Therefore length is $\frac{3}{2}x$.




$\left(\frac{3}{2}x + 4\right)(x+4) - x\left(\frac{3}{2}x\right) = 116$

$6x + 4x + 16 = 116$

$10x = 100$

width $x = 10$m, length 15m, area of pool$= 150$m$^2$.

### R2

(a) $x(y+3) = y$

$y - xy = 3x$

$y(1-x) = 3x$

$y = \frac{3x}{1-x}$

(b) $\frac{1}{y} = \frac{1}{x} - \frac{1}{z}$

$\frac{1}{y} = \frac{z-x}{xz}$

$y = \frac{xz}{z-x}$

(c) $y^2(a-b) = 4$

$y = \pm\sqrt{\frac{4}{a-b}} = \pm\frac{2}{\sqrt{a-b}}$

(d) $x^2 = y^2 - 4ab$

$y^2 = x^2 + 4ab$

$y = \pm\sqrt{x^2 + 4ab}$

### R3

(a)

(i) $x^2 = \frac{3V}{h}$

$x = \sqrt{\frac{3V}{h}}$, but note $x > 0$

(ii) $x = \sqrt{\frac{3 \times 15}{5}} = 3$ cm

(b)

(i) $r^2 = \frac{V}{\pi h}$

$r = \sqrt{\frac{V}{\pi h}}$

(ii) $r = \sqrt{\frac{100}{4\pi^2}} = \frac{5}{\pi}$ cm

(c)

(i) $r^3 = \frac{3V}{4\pi}$

$r = \sqrt[3]{\frac{3V}{4\pi}}$

(ii) $r = \sqrt{3 \times \frac{9\pi}{2} \times \frac{1}{4\pi}}$

$= \frac{3}{2}$ cm.

### R4

(a) Let the length be $x$ cm, then the width is $x - 6$ cm.

$P = 60 = 4x - 12$

$4x = 72$

$x = 18$ cm

$\therefore$ length of rectangle is 18 cm

(b) Let Emma's age be $x$ years, therefore Robin's age is $x + 23$ years. In 6 years
$x + 23 + 6 = 2(x + 6)$
$x = 17$
Emma is 17 and Robin is 40 years old.

**R5**

(a) $(x - 10)(x + 1)$

(b) $(x + 9)(x - 7)$

(c) $(x - 2)(5x - 4)$

(d) $(5x + 3)(x - 1)$

(e) $(4 + x)(5 - 2x)$

(f) $5(x - 5) + x(x - 5) = (x - 5)(5 + x)$

(g) $(5 + x - 1)(5 - (x - 1))$ difference of squares
$= (4 + x)(6 - x)$

(h) $(x - y)(x + y) + 3(x - y) = (x - y)(x + y + 3)$

(i) $(x + y + 2)(x + y - 2)$

(j) $5(16x^4 - y^4) = 5(4x^2 - y^2)(4x^2 + y^2)$
$= 5(2x - y)(2x + y)(4x^2 + y^2)$

(k) $x^2(x - 3) - 4(x - 3) = (x - 3)(x - 2)(x + 2)$

(l) $(2x - 3y)(x + y)$

**R6**

(a) $\sqrt{2} + 2\sqrt{2} = 3\sqrt{2}$

(b) $2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$

(c) $2\sqrt{5} + 3\sqrt{5} = 5\sqrt{5}$

(d) $3\sqrt{10} - 2\sqrt{10} = \sqrt{10}$

(e) $4 - 12\sqrt{2} + 18 = 22 - 12\sqrt{2}$

(f) $6 - 3 = 3$

**R7**

(a) $3 + 1 = 4$

(b) $a^{-1} = \dfrac{1}{a}$

(c) $9x^4 \div 6x^4 = \dfrac{3}{2}$

(d) $\dfrac{3^{2m-2} \times 2^{m+3}}{3^{m+2} \times 2^{m+2}} = 2 \times 3^{m-4}$

**R8**

(a) $\left(8^{\frac{1}{3}}\right)^2 = 2^2 = 4$

(b) $\left(25^{-\frac{1}{2}}\right)^3 = \left(\dfrac{1}{5}\right)^3 = \dfrac{1}{125}$

(c) $\left[\left(\dfrac{1}{16}\right)^{-\frac{1}{4}}\right]^3 = 2^3 = 8$

**R9**

5 lots of $5^5 = 5 \times 5^5 = 5^6$

**R10**

(a) $2^{x+3} = 2^4$
$x = 1$

(b) $2^{3x} = 2^{-2}$
$x = -\dfrac{2}{3}$

(c) $2^{2x} = 2^{\frac{1}{2}}$
$x = \dfrac{1}{4}$

(d) $3^{2x-1} = 3^0$
$x = \dfrac{1}{2}$

(e) $4 + 3^x = 36$
$3^x = 9$
$x = 2$

(f) $2^{2x} - 5 \times 2^x + 4 = 0$
$(2^x - 4)(2^x - 1) = 0$
$x = 2 \quad x = 0$

**R11**

(a) $\sqrt{2} + 1$

(b) $\dfrac{(2 + \sqrt{5})^2}{4 - 5} = \dfrac{9 + 2\sqrt{5}}{-1}$

(c) $\dfrac{(4 + \sqrt{2})(1 - \sqrt{2})}{-1}$
$= -(2 - 3\sqrt{2})$
$= 3\sqrt{2} - 2$

(d) $\dfrac{3(\sqrt{6} + 2)}{2}$

**R12**

(a) $(2 - \sqrt{3})^2 = a - 4\sqrt{b}$
$= 7 - 4\sqrt{3}$
$a = 7 \quad b = 3$

(b) $\dfrac{(\sqrt{5} - \sqrt{3})^2}{2} = a - \sqrt{b}$
$\dfrac{8 - 2\sqrt{15}}{2} = 4 - \sqrt{15}$
$a = 4 \quad b = 15$

**R13**

(a) $8x - 32 - 12 + 3x = 2x$

$9x = -4$

$x = \frac{44}{9}$

(b) $4x - 6 = 3x - 1$

$x = 5$

(c) $9x - 3 - 4x - 2 = -30$

$5x = -25$

$x = -5$

(d) $-4x^2 + 2x - 6x + 3 = -4x^2 - 2 + 9x$

$13x = 5$

$x = \frac{5}{13}$

**R14**

(a) $\frac{4x - 6 - 3x - 1}{10} = \frac{x - 7}{10}$

(b) $\frac{3}{x - y}$

(c) $\frac{4 + x}{x^2 - 4}$

(d) $\frac{x(x + y) - y}{x^2 - y^2} = \frac{x^2 + xy - y}{x^2 - y^2}$

(e) $3\left[\frac{x - 1 - x - 1}{(x + 1)(x - 1)}\right] = \frac{-6}{(x + 1)(x - 1)}$

(f) $\frac{2(x + 7) - 3(x + 2)}{(x - 2)(x + 2)(x + 7)}$

$= \frac{-x + 8}{(x - 2)(x + 2)(x + 7)}$

(g) $\frac{x^2 - y^2 - x^2}{x + y} = \frac{-y^2}{x + y}$

(h) $\frac{4 - (x + 2) + 5(x - 2)}{x^2 - 4}$

$= \frac{4x - 8}{(x + 2)(x - 2)} = \frac{4}{x + 2}$

(i) $\frac{x + 3 - x}{(x + 3)^2} = \frac{3}{(x + 3)^2}$

(j) $\frac{1 - (x + 1)}{(x + 1)^2} = \frac{-x}{(x + 1)^2}$

(k) $\frac{1}{\frac{1}{x} + \frac{1}{y}} = \frac{xy}{y + x}$

(l) $\frac{1}{x + y}\left(\frac{1}{x} + \frac{1}{y}\right) = \frac{1}{xy}$

**R15**

(a) $x + \frac{1}{x} - 2$

(b) $x - \frac{1}{x}$

**R16**

(a) $a + \frac{1}{a} = 2 + \sqrt{3} + \frac{1}{2 + \sqrt{3}} \times \frac{2 - \sqrt{3}}{2 - \sqrt{3}}$

$= 2 + \sqrt{3} + 2 - \sqrt{3}$

$= 4$

(b) $\left(a + \frac{1}{a}\right)^2 = a^2 + 2 + \frac{1}{a^2}$

$(4)^2 = a^2 + 2 + \frac{1}{a^2}$ using result (a)

$\therefore \; a^2 + \frac{1}{a^2} = 16 - 2$

$= 14$

**R17**

(a) $y^2$

(b) $y + y^2$

(c) $y^3$

(d) $4y^2$

(e) $2x^6$

(f) $8x^6$

(g) $a^{m+n}$

(h) $1$

(i) $1 + \frac{y}{x}$

(j) $\frac{1}{y} + \frac{1}{x}$

(k) $1 + y$

(l) $2x + 3y$

(m) $2 + \frac{3y}{x}$

(n) $\frac{x + y}{x} = 1 + \frac{y}{x}$

(o) $\frac{2xy}{1 + 2y}$

(p) $\frac{5x}{4}$

(q) $\frac{x + y}{x^2y^2}$

(r) $\frac{y + x}{y - x}$

(s) $\frac{2}{x + y}$

(t) $-1$